1

Plurality

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1.1 Introduction

As far as morphology is concerned, the notion of plurality is quite straightforward: Plurality is an instance of the inflectional category of number. The nominal form *piano* is singular, while *pianos* is plural. From a semantic point of view, the concept of plurality is much more diffuse. To start with, the general folk linguistic intuition that being plural expresses being *more than one* is inaccurate. For example, while (1-a) suggests there being multiple stains on the carpet, (1-b) does not suggest that only the carrying of *multiple* guns is illegal, nor does (1-c) suggest the possibility of a single alien having walked the earth.

(1) a. There are stains on the carpet.
    b. Carrying guns is illegal in Illinois.
    c. No aliens have ever walked the earth.

Furthermore, plurality may emerge from sentences that lack any plural morphology, such as the first sentence in (2), which expresses how some number of boys drew pictures. This emergent plurality becomes apparent in the continuation in which a plural pronoun refer to the pictures that the boys drew.

(2) Each boy drew a picture. They are hanging on the wall.

It is sometimes thought that, in languages like English (and indeed Indo-European languages, generally), plurality is an essentially nominal phenomenon. Corbett 2000, for instance, uses (3) to point out that the plural verbal form must be an uninterpreted agreement reflex triggered by the (unmarked) plurality of the noun. We cannot for instance use (3) to express that a single sheep drinks from the stream more than once. It has to mean that there were multiple sheep.

(3) The sheep drink from the stream. [?]

In other languages number marking may be verbally expressed. For instance, (4), an example from Hausa from [?], is incompatible with their being a single kick to the table, and typically expresses a quick repetition of kicks. This interpretation is triggered by partial reduplication of the verb stem.[1]

(4) Yaa shùs-shùuri teebùr̂
3SG.MASC REDUP-kick table
'*He kicked the table repeatedly*'

The observation that, in English, plural marking has obvious semantic effects in the nominal, but not in the verbal domain, should not be mistaken for a deeper semantic conclusion, namely that there is nothing to study beyond plural reference. In fact, it turns out that the most puzzling data involving plurality concern exactly the interaction between plural arguments and verbs. This can be illustrated by the following observation: predicates differ with respect to the entailment patterns they display for plural arguments. Take the *distributivity* entailment pattern in (5).

(5) A predicate "VP" is *distributive* if and only if "X and Y VP" entails "X VP" and "Y VP"

Being wounded is an example of a predicate that clearly supports the entailment scheme in (5), for (6-a) entails both (6-b) and (6-c).

(6) a. Bob and Carl are wounded.
b. Bob is wounded.
c. Carl is wounded.

For other predicates, the pattern in (5) does not hold. Consider, the following examples.

(7) a. Bob and Carl carried a piano up the stairs.

[1] Plurality involving the multiplicity of events and some kind of verbal morphological marking is often referred to a *pluractionality* [?]. One difficult question is how (or whether, Corbett 2000) pluractionality differs from aspectual marking or Aktionsart. See Wood 2007 for discussion. In this chapter, I will ignore the phenomena of pluractionality.

b. Bob carried a piano up the stairs.
c. Carl carried a piano up the stairs.

In a situation in which both (7-b) and (7-c) are true, (7-a) is true too. However, (7-a) is also true in a situation in which Bob and Carl carried the piano up in a joint effort. In that case, it would be false to say (7-b) or (7-c). It turns out then that whilst *to carry a piano up the stairs* is not distributive, it does have the inverse property:

(8) A predicate "VP" is *cumulative* if and only if "X VP" and "Y VP" entails "X and Y VP"

Note that *being wounded*, besides begin distributive, is cumulative, too. It follows from (6-b) and (6-c) that (6-a). Yet other predicates are neither distributive nor cumulative.

(9) a. Bob and Carl are a couple.
b. *Bob is a couple.
c. *Carl is a couple.

In sum, it depends on the predicate what kind of entailment patterns we can observe for plural arguments. Plurality will therefore need to be studied in a compositional context. It is this interaction between plural arguments and predicates that is the focus of this chapter. The structure is as follows. In section 1.2, I will introduce the most commonly assumed complication that plurality brings to the logical language, namely the inclusion of logical constants that are not atomic but *sums*. Using such plural individuals, I turn to the distributive entailments, as well as distributive readings of sentences in **??**. Section **??** turns to a special class of interpretations for sentences with plurality, namely cumulativity and its analysis in event semantics. In section **??**, I briefly look at the relation between quantifiers and distributivity. Finally, in section **??**, I discuss the relation between the readings of plural sentences and anaphora.

1.2 Sums

Let us consider the simple statement in (10) again.

(10) Bob and Carl are wounded.

Given the observed distributivity entailment of *being wounded*, it is tempting to analyse (10) as (11). Not only does (11) yield the intuitively correct truth-conditions, it does so without complicating the predicate-logical language. That is, the plurality in natural language is *distributed* over several conjuncts of essentially singular predications.

(11) $wounded(b) \wedge wounded(c)$

The discussion on entailment patterns above, however, shows that this kind of analysis is not very promising, for it would fail on predicates that lack distributive entailments. For instance, analysing (12-a) as (12-b) incorrectly predicts that (12-a) is false on a scenario where Bob and Carl collectively carry a piano up the stairs.

(12) a. Bob and Carl carried a piano up the stairs.
b. $[\![\text{carried a piano up the stairs}]\!]\,(b)$
$\wedge\,[\![\text{carried a piano up the stairs}]\!]\,(c)$

The common solution is to assume that the phrase *Bob and Carl* refers to a plural individual and that this individual is, as a single collective individual, the argument of the predicate. It would not do, however, to just assume that the conjunction Bob and Carl is a name for some plural individual. There must be some connection between the plural individual that *Bob and Carl* refers to and the individual entities Bob and Carl. This is because, ultimately, we want an account of how certain predicates allow for distributivity and/or cumulativity entailments that link statements involving plural arguments to statements involving the singular parts. Here we can just assume that pluralities are to singularities the way sets are to their members. That is, if D_e is the set of singular entities, then the set of plural individuals will be the powerset of this domain with the empty set removed: $\wp(D_e) \setminus \emptyset$. We can now say that *Bob and Carl* refers to the smallest set that has Bob and Carl as members. So, for (12-a) we now get (13).

(13) $[\![\text{carried a piano up the stairs}]\!]\,(\{b, c\})$

While there exist analyses along this line (Hoeksema, Schwarzschild, Winter), a more common alternative is to use an approach inspired by the work of Link (1982, 83, see 98 for collection, and alterations by Landman ?? @@@ divide parsons, gillon loenning and lasersohn over these references @@). In this approach, there is no type-theoretic difference between plural and singular individuals. So, plural individuals are e-type entities just like singular individuals are. However, the domain of entities is structured exactly like a powerset of the set of atomic individuals.

We can set this up as follows. Let D_e be the set of all (i.e. both singular and plural) entities and let $\sqcup$ be a binary operation on elements of D_e, called *summation*. Summation has the following properties:

(14) a. $(\alpha \sqcup \beta) \sqcup \gamma = \alpha \sqcup (\beta \sqcup \gamma)$ associative
b. $\alpha \sqcup \beta = \beta \sqcup \alpha$ commutative
c. $\alpha \sqcup \alpha = \alpha$ idempotent

Given summation, we now define a partial order $\leq$ on D_e:

(15) $\alpha \leq \beta$ if and only if $\alpha \sqcup \beta = \beta$

The idea is that just like $\{b, c\}$ is the smallest set that contains b and c as elements, so is $b \sqcup c$ – i.e. the *sum* of b and c – the smallest entity that has b and c as its parts. This means that $\sqcup$ is a supremum or *join* operator. The desired structure is a particular kind of lattice, namely a complete atomic join semi-lattice (see Link 1998, chapter 6 and Landman 1991 for details). *Complete* since we want the domain of entities to be closed under $\sqcup$; *atomic* since we also want that all atomic parts of sums in the domain are part of the domain themselves. (If we can talk about *Bob and Carl*, then we also can talk about *Bob*.) Finally, it is a *semi*-lattice, since we are only interested in joins, not in meets.

In (16), I give a few handy functions and predicates inspired by Link's work. First of all (16-a) says that something is an atomic entity if and only if it only has itself as a part. The operation in (16-b) restricts a set of individuals to the ones that are atomic.

(16) a. $Atom(\alpha)$ if and only $\forall \beta \leq \alpha[\alpha = \beta]$
b. $Atoms(A) = \lambda \alpha . \alpha \in A \wedge Atom(\alpha)$

Given these definitions, we can formally express the correspondence between the lattice structure assumed by Link and the powerset structure we were aspiring to:

(17) $\langle D_e, \sqcup \rangle$ is isomorphic to $\langle \wp(Atoms(D_e)) \setminus \emptyset, \cup \rangle$

The operator most borrowed from Link's work is the sum closure operator $\sqcup$ for sets or one-place predicates:

(18) $^*X$ is the smallest set such that:

$$*X \supseteq X \text{ and } \forall x, y \in *X : x \sqcup y \in *X$$

With the $\sqcup$-operation defined, we can start to think about what plural DPs refer to. In line with the idea of sums as supremums, it would make sense to think of the conjunction *Bob and Carl* as referring to $b \sqcup c$: the individual that is made up of Bob and Carl only.

Link reasoned that while singular nouns referred to sets of atoms (i.e. elements in $\wp(Atoms(D_e))$), plural nouns refer to sets that include plural individuals. (We will ignore for now the hairy issue whether or not such plural nouns also contain atomic entities in their extension. See @@ for discussion.) So:

(19) $[\![boys]\!] = *[\![boy]\!]$

So, if $[\![boy]\!]$ is the set $\{b, c, d\}$, then $[\![boys]\!]$ is the set $\{b, c, d, b \sqcup c, c \sqcup d, b \sqcup d, b \sqcup c \sqcup d\}$. So, (20-a) is now predicted to be true, by virtue of the fact that the plural individual $b \sqcup c$ is in the pluralised – i.e. $*$-ed – extension of the noun *boy*, which we could express in a predicate logical language as in (20-b).

(20) a. Bob and Carl are boys.
b. $*boy(b \sqcup c)$

The question is now how to account for predicates with different entailment patters. For this, we need to have a closer look at *distributivity*.

1.3 Distributivity

The sentence in (21) is true in a number of quite different scenarios. In a *collective* understanding of the sentence, Bob, Carl and Dirk jointly carry a piano up the stairs. Conversely on a *distributive* understanding Bob carried a piano up the stairs by himself, and so did Carl, and so did Dirk.

(21) Bob, Carl and Dirk carried a piano up the stairs.

The distributive understanding can be isolated by adding a floating distributive quantifier like *each*: that is, *Bob, Carl and Dirk each carried a piano up the stairs* only has the distributive reading. With this in mind, predicates with distributive entailments are always understood distributively, since for (22-a) the adding of *each* in (22-b) seems vacuous.

(22) a. Bob, Carl and Dirk are wounded.
b. Bob, Carl and Dirk are each wounded.

In summary, we get the following overview:

(23)

	entailments		-	understandings	
	distr	cumul	-	distr	collec
being wounded	yes	yes	-	yes	no
to carry a piano up x	no	yes	-	yes	yes

The stable factor in this table is that all examples allow for cumulative inferences and all examples have distributive understandings. Note that Link's *-operation amounts to enforcing cumulative entailments. That is, if predicates with plural arguments are pluralised, then cumulativity follows, due to the following fact:

(24) $P(\alpha) \wedge P(\beta) \Rightarrow {}^*P(\alpha \sqcup \beta)$

The inverse entailment does not follow, for ${}^*P(\alpha \sqcup \beta)$ could be the case without $P(\alpha)$ being the case. Just take a P such that $P(\alpha \sqcup \beta)$ is true but not $P(\alpha)$.

Landman (@which@ again) proposes to account for the difference between predicates like *being wounded* and predicate like *to carry a piano up the stairs* by not just assuming that predicates with plural arguments are plural, but by furthermore assuming that predicates differ with respect to the kind of entities they have in their (singular) extension. For *being wounded* it suffices to assume that its extension only contains atoms. This coincides with the intuition that it does not make sense for the property of being wounded to be shared among multiple entities. We can now analyse (22-a) as (25).

(25) ${}^*wounded(b \sqcup c \sqcup d)$

The cumulativity entailment follows from (25). The inverse distributivity entailment follows from the fact that the only way a plurality can end up in the pluralised extension of the predicate is if its parts were in the singular extension. Another way of saying this is that there cannot be any collective understandings. All plural arguments are the result of the *-operation.

Adopting a similar strategy for (21) results in (26).

(26) $^* [\![\text{carried a piano up the stairs}]\!] \, (b \sqcup c \sqcup d)$

The key to accounting for the properties of (26) is to assume that the extension of predicates like *to carry a piano up the stairs* might or might not include atoms and it might or might not include sums; that this fully depends on the state of affairs. In a distributive scenario, b, c and d occur in the extension as singular individuals. Given the effect of pluralisation (26) is true in that scenario. In the collective scenario the extension just contains $b \sqcup c \sqcup d$ as a plurality, and hence (26-a) is also true. There is lack of distributivity entailment, since we cannot know which scenario made (26-a) true.

Contrary to what the analysis above suggests, it is sometimes assumed that what I have vaguely called *understandings* are in fact different *readings* of an ambiguity. On such accounts, (21) is ambiguous between a reading in which the distributivity holds (the distributive reading) and one in which it does not (the collective one). The latter reading is more basic, while the distributive one is derived using the insertion of a distributivity operator. There exist two main variations on such approaches, depending whether the operator in question is applied to the verb's argument or to the verb itself. Take, for instance, the following two options.

(27) a. $DIST = \lambda\alpha.\lambda P.\forall\beta \leq \alpha[Atom(\beta) \rightarrow P(\beta)]$
b. $DIST = \lambda P.\lambda\alpha.\forall\beta \leq \alpha[Atom(\beta) \rightarrow P(\beta)]$

In a case like (21), insertion of DIST will result in a reading that carries a distributivity entailment: $\forall x \leq b \sqcup c \sqcup d[Atom(x) \rightarrow [\![\text{carried a piano up the stairs}]\!](x)]$. Hence, this reading is exactly what one would get by adding a floating *each* to (21). For cases like (21) the choice between (27-a) and (27-b) is immaterial. It is easy to find examples, however, where these two operators differ in predictions. Following Lasersohn 1989, a sentence like (28-a) illustrates this. Here, it is possible for one part of the conjoined verb phrase to get a collective reading whilst the other part gets a distributive reading. Using (27-b) this is easy to capture, as in (28-b). However, no easy analysis seems to be available using (27-a).

(28) a. Bob and Carl carried a piano up the stairs and then drank a glass of water.
b. $$\left[\begin{array}{ll}\lambda x. & [\![\text{carried a piano up the stairs}]\!]\,(x) \wedge \\ & DIST(\,[\![\text{drank a glass of water}]\!]\,)(x)\end{array}\right](b \sqcup c)$$

Assuming we should abandon the option in (27-a), we now have two competing accounts of distributivity. These are definitely closely related. In fact, it is easy to see that the following correspondence holds:

(29) $DIST(A) = {}^*(Atoms(A))$

Either distributivity is not a reading at all and it is just an inference triggered by pluralisation (mainly Landman 1996, 2000), or it is due to the optional insertion of a distributivity operator, and thereby an independent interpretation by itself.

There exists some evidence against this latter strategy. Schwarzschild 1996 uses one of the classic ambiguity tests from Zwicky and Sadock (1975). Whenever an ambiguous term is elided, then it has to be resolved to the same interpretation as its antecedent. For instance, (30-a) can mean that the speaker shot an elephant that was wearing pyjamas and that the speaker's sister shot a similar animal or it means that the speaker shot an elephant whilst wearing pyjamas and that his or her sister shot an elephant in similar attire. Crucially, it cannot mean that the speaker shot an elephant that was wearing pyjamas, whilst the sister shot an elephant while she was wearing pyjamas. Schwarzschild now observes that for sentences like (30-b) such a mixed interpretation is perfectly possible. For instance, (30-b) is intuitively true if each of the computers were paid in two instalments and the entire collection of diskettes were collectively paid in two instalments. (See Schwarzschild for a more elaborate scenario that makes this understanding likely.)

(30) a. I shot an elephant wearing pyjamas and my sister did too.
b. The computers were paid in two instalments and the diskettes were too.

While Schwarzschild's observation strongly suggests that we should not really be talking about *readings*, there is some data that seems to indicate that plural sentences at least involve some covert operator. also some indications that things might be slightly more complicated. One reason to believe that this is the case comes from Heim et al. 1991. Consider (31).

(31) Bob and Carl think they will win €1000.

This example has multiple readings depending on how the pronoun *they* is resolved and whether the verb phrases are construed distributively or collectively. Three of those are in (32).

(32) a. Bob and Carl argue that Bob and Carl, as a group, will win €1000.
b. Bob argues that Bob will win €1000 and Carl argues that Carl will win €1000.
c. Bob argues that Bob and Carl, as a group, will win €1000 and Carl argues the same.

In (32-a), the plural pronoun *they* is understood as co-referring to the matrix subject *Bob and Carl*. This is a fully expected reading. However, the reading in (32-b) is not so trivial to account for. In this case, the pronoun is interpreted as a bound variable. But that means that there has to be some quantificational operator.

While these data are usually thought to indicate the presence of a proper distributivity operator akin the natural language floating *each* in distributive readings, the observations are not incompatible with a theory in which a single operator accounts for a range of understandings, including Landman's use of *. Also, Schwarzschild 1996 offers an analysis that uses a distributivity operator, without predicting that there are multiple readings. In a nutshell, and simplifying drastically, the idea is that pluralities can be covered, i.e. divided up into parts and that the distributivity operator quantifies over such parts.[2] A definition of *minimal sum covers* as well as some examples are (33), while (34) defined the distributivity operator.

(33) C minimally covers α iff $^*C = \alpha$ and $\neg\exists C' \subset C[^* * C' = \alpha]$
a. $\{b, c \sqcup d\}$ minimally covers $b \sqcup c \sqcup d$
b. $\{b, c, d\}$ minimally covers $b \sqcup c \sqcup d$
c. $\{b \sqcup c \sqcup d\}$ minimally covers $b \sqcup c \sqcup d$

(34) $DIST(P) = \lambda x.\forall y \in C_x[P(y)]$
where C_x is some pragmatically determined minimal cover of x

This one definition covers a range of readings, depending on what kind of cover is pragmatically given. For instance, (35-a) will be interpreted as (35-b).

(35) a. These six eggs costs €2.
b. $DIST(\lambda x.cost(x, €2))$(the-6-eggs)

[2] See [**?**] for a more advanced integration of Schwarzschild's idea in a sum-based approach to plurality. The idea of using *covers* to account for different understandings of plural statements originates in Gillon 1987.

On its most salient reading, the relevant cover would be a cover that does not divide the eggs up, but one which just offers a single cell containing the six eggs. This results in a collective understanding. The distributive reading (where these are particularly expensive eggs) results from taking a cover that divides the eggs up into their singular parts such that (35-b) ends up expressing that each egg cost two euros. There are other theoretical possibilities, which are however not so salient for the example in (35-a). For an example like (36), however, the relevant cover would be one in which the shoes are divided up in pairs, since, pragmatically, that is how shoes are normally valuated.

(36) The shoes cost $75 Lasersohn 2006

So far, I have discussed distributive and collective understandings of plural predication, as well as intermediate ones as in (36). I now turn to yet another understanding, *cumulativity*.

1.4 Cumulativity

In the discussion of distributivity above, we saw that one approach (basically that of Landman 1996, 2000) involves reducing distributivity to pluralisation of $\langle e, t\rangle$ predicates, using Link's *-operation. Notice, that in the examples so far we have always *-ed the VP denotation. This is not the only theoretical option. Consider (37).

(37) Two boys carried three pianos up the stairs.

We could imagine quantifier raising the object and thereby creating a derived predicate 'λx.two boys carried x up the stairs'.[3] Using the *-operation, we would then expect there to exist a distributive reading for the object: there are three pianos such that each of these pianos was carried up the stairs by a potentially different pair of boys. This reading is available to some, but certainly not all, speakers of English. While there may be linguistic reasons to limit such object distributivity, it shows that distributivity is tightly linked to scope.

One way to make the relevant scopal relations more visible is by counting the number of entities that are involved in a verifying scenario. To do this reliably, let me change the example to (38). (A crucial difference to (37) is that a book cannot be written twice.)

[3] As would be standard in, say, the textbook account of Heim & Kratzer 1998.

(38) Two Dutch authors wrote three gothic novels.

A collective understanding of (38) is verified by a group of two Dutch authors who collectively wrote three gothic novels. On a distributive reading the verifying situations involve six gothic novels, three for each Dutch author. It is dependency of the number of books on the number of authors that shows that the subject has scope over the object. Using the number of entities involved as diagnostic of scope shows us that there another kind of understanding, on top of the distributive and the collective one. This is the so-called *cumulative reading* [**?**]. For instance, (39) (a variation on an example from Landman 2000), is true in a situation in which there were (in total) six frog swallowing ladybirds and there were (in total) twelve ladybirds being swallowed by frogs.[4]

(39) Six frogs swallowed twelve ladybirds.

The reported reading cannot be a collective reading, since *swallow* is a predicate with distributive entailments. (It makes no sense to swallow a single entity in a joint effort.) It also clearly is not a distributive reading, since the number of ladybirds is twelve, and not seventy-two.

Similarly, (38) has cumulative reading in which there were two Dutch authors writing gothic novels and there were three gothic novel written by Dutch authors. Again, this is not the collective reading. Take for instance, a verifying situation like one in which one Dutch author writes a gothic novel by himself and then co-writes two more gothic novels with another Dutch author.

To set up the popular account of cumulativity, let us go through an elaborate illustration of (38). Assume the following semantics for the subject and the object respectively.

(40) a. $\lambda P.\exists x[\#x = 2 \wedge {}^*dutch.author(x) \wedge P(x)]$
b. $\lambda P.\exists y[\#y = 3 \wedge {}^*gothic.novel(y) \wedge P(y)]$

Here I use # for the 'cardinality' of a sum, as given by the following definition.

(41) $\#\alpha := |Atoms(\lambda\beta.\beta \leq \alpha)|$

The (subject) distributive reading is now given by pluralising the verb phrase:

[4] Landman 2000 uses a similar example to argue against the reduction of cumulative readings to collective reading, e.g. [**?**].

(42) $\exists x[\#x = 2\wedge {}^*dutch.author(x)\wedge {}^*\lambda x'.\exists y[\#y = 3\wedge {}^*gothic.novel(y)\wedge wrote(x', y)](x)$

This is not entirely accurate, since this assumes that the object is interpreted collectively. (It is not entirely clear how books can be collectively written.) So, a probably more accurate semantics is a doubly distributive reading, as in (43):

(43) $\exists x[\#x = 2\wedge {}^*dutch.author(x)\wedge {}^*\lambda x'.\exists y[\#y = 3\wedge {}^*gothic.novel(y)\wedge {}^*\lambda y'.wrote(x', y')(y)](x)$

The cumulative reading results not from pluralising a derived predicate, but from directly pluralising the binary relation *write* (Krifka 1989, 1992, Landman 1996, 2000). That is, we need to generalise the *-operator to many-placed predicates. Let X be a set of pairs:[5]

(44) $\langle\alpha, \beta\rangle \sqcup \langle\gamma, \delta\rangle := \langle\alpha \sqcup \gamma, \beta \sqcup \delta\rangle$

Now the *-operator can be generalised to binary relations.

(45) ${}^*X$ is the smallest set such that ${}^*X \subseteq X$ and $\forall x, x' \in {}^*X : x \sqcup x' \in {}^*X$

The cumulative reading of (38) is now simply (46).

(46) $\exists x[\#x = 2 \wedge {}^*dutch.author(x) \wedge \exists y[\#y = 3 \wedge {}^*gothic.novel(y) \wedge {}^*wrote(x, y)]]$

For instance, if the extension of *wrote* is $\{\langle b, gn1\rangle, \langle b \sqcup c, gn2\rangle, \langle b \sqcup c, gn3\rangle\}$, then ${}^*wrote$ will contain $\langle b \sqcup c, gn1 \sqcup gn2 \sqcup gn3\rangle$, verifying (38).

There are two well-studied problems with this account. First of all, because the cumulative reading is scopeless, the above account only works for examples with commutative pairs of quantifiers, such as the two existentials in (46). For instance, (47) also has a cumulative reading, but on a standard generalised quantifier approach to modified numerals, as in (48), the quantifiers are not commutative, since they count atoms and thereby distribute over their scope.

(47) In 2011, more than 100 Dutch authors wrote more than 200 gothic novels.

[5] More general, for n-ary tuples f, g: $f \sqcup g :=$ that n-ary tuple h, such that $\forall 1 \leq m \leq n : h(m) = f(m) \sqcup g(m)$.

(48) $[\![\text{more than } n]\!] = \lambda P.\lambda Q.|\lambda x.P(x) \wedge Q(x)| > n$

Krifka's solution is to analyse numerals as existential quantifiers (as in (40)) and their modifiers as propositional operators on alternative numerical values. So, the interpretation of (47) yields the set of cumulative propositions in (49). The role of the numeral modifiers is now to assert that there is a factual proposition in this set for $n > 100$ and $m > 200$.[6]

(49) $$\left\{ \begin{array}{l|l} \exists x[\#x = n \wedge {}^*dutch.author(x) \wedge & \\ \exists y[\#y = m \wedge {}^*gothic.novel(y) \wedge & n, m \in \mathbb{N} \\ {}^*wrote(x, y)]] & \end{array} \right\}$$

A second problem with the particular pluralisation account of cumulativity comes from examples where a cumulative reading is combined with a distributive one. Such examples were divised by Schein (199?). The standard example is (50). Crucially, there is cumulativity with respect to the three cash machines and the two new members, but the passwords are distributed over these two.

(50) Three cash machines gave two new members each exactly two passwords.

Since each new member was given two passwords, *two new members* has to have scope over *two passwords*. But that scoping should be independent of the scopeless relation between the cash machines and the members. Schein uses examples like (50) as part of an elaborate argument that a plural semantics should be part of a neo-Davidsonian event semantics, where events have a similar part-whole structure as entities. Partly due to this argument events play a major role in the dominating approaches to plurality. See, for instance, [**?**] for recent discussion and [**?**] for a criticism of some of the arguments for the neo-Davidsonian approach. In the interest of brevity, I refrain from discussing events in this chapter.

1.5 Distributivity and dependency

Since, unlike cumulative readings, distributive readings are scopal, transitive distributive sentences may introduce a dependency between the

[6] An alternative account is to treat numeral modification as a degree phenomenon. See, for instance, [**?**, **?**].

arguments of the verb. For instance, on the distributive reading, (51) expresses a relation between three boys and (at least) three essays.

(51) Three boys wrote an essay.

This dependency becomes grammatically relevant in at least two different ways. First of all, in a number of languages, indefinites that are dependent in the way *an essay* is in (51) have to be marked as such. (For instance, see Farkas 1997 for Hungarian, Farkas 2002 for Romanian, Yanovich 2005 for Russian, and Henderson 2011 for the Mayan language Kaqchikel). Dubbed *dependent indefinites* by Farkas, such indefinites come with a constraint that they have to co-vary with some other variable.

(52) A gyerekek hoztak egy-egy k onyvet
the children brought a-a book.ACC
'The children brought a book each.'

Another form of dependency involves anaphora.[7]

(53) a. Three students wrote an article. They sent it to L&P.Krifka 1996
b. John bought a gift for every girl in his class and asked their desk mates to wrap them. after Brasoveanu 2008

The example in (53-a) has many readings. The most salient one is the one in which both sentences are distributive. On that reading, the first sentence introduces a dependency between students and articles, which is then accessed by the pronouns in the second sentence. That is, the second sentence can only mean that each student sent *his or her* paper to L&P. Similarly, there is a salient reading for (53-b) in which for each girl, John asked the desk mate of that girl to wrap the gift he bought for *her*.

Such data are hard to account for, especially given some other complicating features of anaphora. For instance, dependent indefinites are antecedents of plural, not singular, pronouns, which have maximal reference. In an example like (54), the singular pronoun is infelicitous and the plural pronoun is interpreted as referring to the set of all essay written by a student.

[7] See Winter 2000 for an attempt at reducing cumulativity to distributivity and dependency and Beck and Sauerland for a criticism of that account.

(54) Several students wrote an essay. I will need to grade them / *it before tomorrow.

In contrast, to (54), singular anaphoric reference to a dependent indefinite is possible once the pronoun is in the same kind of dependency environment, as shown by the examples in (53).

A powerful account of these facts is offered by the dynamic mechanisms proposed in van den Berg (1996) and subsequent related work [**?**, **?**, **?**, **?**].[8]

To get the idea behind these proposal, first consider the simple example in (55).

(55) Two boys were wounded. They are in hospital.

Under a dynamic semantic approach to anaphora, anaphoric relations are captured by assigning antecedent and pronoun the same variable name. For instance, the first sentence in (55) could correspond to $\exists x[\#x = 2 \wedge {}^*boy(x) \wedge {}^*wounded(x)]$. This sentence is verified in a model in which, say, b and c were wounded, by assigning $b \sqcup c$ to x. The second sentence may now be represented by ${}^*inhospital(x)$, where the pronoun automatically referred to whichever two boys are witnesses for the first sentence.

The innovation brought in van den Berg's work is to assume that the assignment of pluralities to variables happens in a much more structured way. So, rather than using the assignment in (56-a), a plurality assigned to a variable, van den Berg uses (56-b), a plurality (set) of assignments to the same variable.

(56) a. $\{\langle x, b \sqcup c\rangle\}$
b. $\{\{\langle x, b\rangle\}, \{\langle x, c\rangle\}\}$

The main reason for doing this is that it opens up a way to represent dependencies created by distributivity. For instance, say that Bob is dating Estelle and Carl is dating Fran. This makes (57) true. But dynamically the sentence will introduce the variable assignment in (57-b), rather than the simplistic (57-a).

(57) Bob and Carl are dating a girl.

[8] See [**?**] and [**?**] for alternative approaches, involving parametrised sums and a representational rule for recovering antecedents respectively. Nouwen 2003 contains a comparison of Krifka's proposal and distributed assignment. See Nouwen 2007 for a (technical) discussion and critique of Kamp & Reyle's approach.

a. $\{\langle x, b \sqcup c\rangle, \langle y, e \sqcup f\rangle\}$
b. $\{\{\langle x, b\rangle, \langle y, e\rangle\}, \{\langle x, c\rangle, \langle y, f\rangle\}\}$

1.6 Conclusion

Space limitations have dictated that this chapter present a somewhat narrow focus on plurality. There are a number of issues I have not been able to pay attention to. In conclusion of this chapter, let me mention two of these.

So far, I have mentioned two sources of distributivity. For instance, some predicates are only compatible with atomic entities in their (singular) extension (*being wounded* was the running example). In some approaches there is a distributivity operator that quantifies over atoms in a sum. A third source is the presence of an inherently distributive quantifier. That is, some quantifiers are *distributive* in the sense that they quantify over atoms only.

(58) A quantifier Q of type $\langle\langle e,t\rangle,t\rangle$ is distributive if and only if for any P of type $\langle e,t\rangle$: $Q(P) \Leftrightarrow Q(\lambda x.P(x) \wedge Atom(x))$.

An indefinite quantifier like *three girls* or the definite *the three girls* is not distributive. An example like (59) allows for collective as well as distributive construals.

(59) Three girls carried a piano up the stairs.

Things are different for quantifiers like *every girl*, which are distributive.[9] In (60), any carrying of pianos that involves joint action is irrelevant. Only if each girl carried a piano by herself is (60) true.

(60) Every girl carried a piano up the stairs.

Similarly distributive is *most girls*:

(61) Most girls carried a piano up the stairs.

If (61) is true, then it is true of most individual girls that they carried a piano by themselves. If the majority of girls jointly carried a piano up

[9] One complication is that *every* does have non-distributive readings, as in:

(i) It took every/each boy to lift the piano. [?]

the stairs, whilst a minority stood by and did nothing, (61) is intuitively false.

Quantifiers that are especially interesting from the perspective of plurality are floating quantifiers like *each* and *all*. Whilst *each* enforces a distributive reading in a sentence, as in (62-a), the presence of *all* allows for both distributive and collective understandings. Yet, *all* is incompatible with certain collective predicates, as illustrated in (62).

(62) a. The boys each carried a piano up the stairs.
b. The boys all carried a piano up the stairs.

(63) *The boys are all a good team.

See [**?**] for an analysis of *all* using Schwarzschild's 1996 cover analysis of distributivity and collectivity.

Whilst the semantics of *each* is obviously closely related to (atom-level) distributivity, the main puzzle posed by *each* is of a compositional nature. This is especially the case for so-called *binominal each* [**?**]. For instance, in (64) the *each* distributes over the atoms in the subject plurality, while it appears to be compositionally related to the object. See Zimmermann 2002 for extensive discussion.

(64) These three Dutch authors wrote five gothic novels each.

Another major issue I have not discussed in this chapter is the interpretation of plural morphology. As I observed at the start, plurals do not generally express *more than one*, for (65) suggests that not even a single alien walked the earth.

(65) No aliens have ever walked the earth.

Krifka 1989 suggests that while plurals are semantically inclusive, that is they consist of both non-atomic and atomic sums, singulars can be taken to necessarily refer to atoms. The idea is now that, under normal circumstances, use of a plural implicates that the more specific contribution a singular would have made was inappropriate, and hence that a non-atomic reference was intended. Cases like (65) are simply cases in which the implicature is somehow suppressed, for instance, because of a downward monotone environment. See Sauerland 2003, Sauerland et al. 2005 and Spector 2007 for a detailed analyses along these lines. Such approaches often break with the intuition that the plural form is marked, while the singular form is unmarked and so we would expect that the plural but not the singular contributes some essential meaning ingredient [?].[10] Farkas and de Swart 2010 formulate an account of the semantics of number marking that is in this spirit of Krifka's suggestion but is at the same time faithful to formal markedness in the singular/plural distinction. See Zweig 2008 for discussion on the relation to dependent bare plurals and for discussion of the complication that an event-based semantics for plurality brings.

[10] Most natural languages mark the plural and leave the singular unmarked (e.g. Greenberg 1966, cf. de Swart & Zwarts 2010 for discussion of exceptions).