Modified Numerals: the Epistemic Effect*

Rick Nouwen

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Except for modified numerals that signal precision, like those of the form *exactly* n, modified numerals are generally compatible with the speaker being ignorant of the exact quantity that is under discussion. For instance:

(1) a. I don't know how many mistakes John found in the manuscript...
b. ...but it's definitely *more than 50*.

In this paper, I will look at epistemic (and related) effects of numerically quantified sentences. What makes studying epistemic aspects of modified numerals very interesting is not the compatibility of, say, (1-b) with speaker ignorance, but rather the fact that there exists a class of modified numerals that is *incompatible* with the speaker having *full knowledge* of the quantity in question. Note, first of all, that *more than 50* is not such a modified numeral: it *is* compatible with epistemic competence, as the example in (2) should suffice to point out.

(2) There were exactly 62 mistakes in the manuscript, so that's *more than 50*.

Things change when we turn from *comparative* modified numerals like *more than 50* to *superlative* modified numerals like *at least 50*. Firstly, there is a striking contrast between (2) and (3).[1]

(3) There were exactly 62 mistakes in the manuscript, #so that's *at least 50*.

A second illustration of the epistemic nature of *at least* is in (4). Even on the assumption that it is normal for a man to know exactly how many children he has, (4-a) would make an unremarkable utterance. On the other hand, (4-b) forces the hearer to assume that the speaker is not your average father. The speaker of (4-b) is ignorant about the number of children he has, making an

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[1]There are contexts in which (3) is acceptable. For instance, A bets B that there are at least 50 mistakes in the manuscript. If it turns out that there were exactly 62 mistakes, then A can use (3) to indicate that as far as s/he is concerned 50 is included in what was indicated with the phrase *at least 50*. Since there are clear meta-linguistic aspects to this way of using (3), it would be best to keep such uses of (3) out of the discussion.

example like this more an utterance of, say, a caricatural rock star or of a sperm donor.

(4) a. I have more than 2 children.
 b. I have at least 3 children.

In the next section, I will characterise the data more precisely, but in order to sketch the relevance of modified numerals to this volume, let me preview one aspect of the data that is very much reminiscent of epistemic indefinites.

German *irgendein* is different from English *some* in that it necessarily signals the ignorance of the speaker with respect to the identity of the referent in question, whilst *some* merely implicates this (e.g. Kratzer and Shimoyama 2002, Aloni and Port 2012).

(5) A student called. *compatible with both ignorance and full knowledge of identity*

(6) Irgendein Student hat angerufen.
 IRGENEIN student has called.
 'Some student called.' *only compatible with ignorance of identity*

This contrast is very similar to that between *at least* modified numerals and comparative modified ones: the latter are compatible with ignorance, whilst superlative modified numerals are incompatible with the speaker not being ignorant. But the parallel does not stop there. The ignorance effect disappears in certain embedded contexts. For instance, modal examples like (7) have a reading containing a deontic free choice effect (Kratzer and Shimoyama 2002; Aloni and Port 2012).

(7) Maria muss irgendeinen Arzt heiraten.
 M. must IRGENDEINEN doctor marry

On one of its readings, this example says that Maria has to marry a doctor and that is doesn't matter which one. In this reading, no reference to the speaker's knowledge state is made. A similar observation can be made for superlative modified numerals.

(8) To get tenure, John has to publish at least 3 books.

This example has a reading in which the speaker has full knowledge of what is required of the hearer: as in (7), the epistemic aspect has vanished. Moreover, (8) resembles a free choice interpretation. It says that you cannot get tenure if you publish fewer than 3 books, but that you do as soon as you have published *any* number of books exceeding 2. (See below for a more nuanced characterisation of this reading.)

The goal of the paper is to explore to what extent an implicature mechanism could be responsible for the epistemic and related effects. I am not the first to explore such a path. However, my main motivation for the particular exploration I undertake in this paper is that existing implicature-based accounts of *at least* (Nilsen 2007; Büring 2008; Cummins and Katsos 2010) draw a parallel between superlative modified numerals and the well-studied ignorance and free choice effects of disjunction (Kamp 1973; Zimmermann 2000; Sauerland 2004; Fox 2006). Such accounts often end up stipulating that superlative modified numerals are in some sense disjunctive. In this paper, I hope to show that a comparison to epistemic indefinites offers a less stipulative implicature-based analyses. The idea is not to think of superlative quantifiers as disjunctions, but as expressions that can be analysed to signal an anti-specificity requirement. As I will argue below, this results in an analysis which builds on the assumption that superlative quantifiers closely resemble epistemic indefinites, both from a semantic and from a pragmatic point of view.

I will work out one such possible analysis and propose a link to the analysis of *algún* by Alonso-Ovalle and Menéndez-Benito 2010. Like my own earlier account (Nouwen 2010a), this analysis naturally explains which modified numerals are epistemic and which ones are not. I have good reasons for seeking an alternative to the scalar account of that 2010 article, for as I explain in there, my analysis failed to make the right predictions for upward entailing superlative quantifiers: in short, it offered a good theory of *at most*, but not of *at least*. Unfortunately, as I will explain, the line I explore in this paper is far from perfect too. In other words, the paper remains explorative and offers no comprehensive account of the semantics and pragmatics of modified numerals.

The paper is structured as follows. In section 1, I discuss the modified numeral data, in particular the epistemic effects and the obviation of epistemic effects in the presence of certain kinds of operators. Section 2 offers my take on an analysis which likens superlative modified numerals to disjunctions, carefully analysing some shortcomings of such an account. Moreover, in section 3, I point out a generalisation missed by an account that compares superlative modified numerals to disjunctive statements. I show that, generally, modified numerals that have epistemic effects as well as obviation effects are formally related to expressions that presuppose anti-specificity. This is compared to the anti-singleton condition proposed for the Spanish epistemic indefinite *algún* in Alonso-Ovalle and Menéndez-Benito 2010 in section 4. The same section moreover shows that the scalar aspect of modified numeral poses a serious complication.

1 Data

In this section, I will present a number of contrasts between comparative modified numerals (*more than n*, *fewer/less than n*) and superlative modified numerals *at least n*, *at most n*) that display the special status of the latter class. I will follow Nouwen (2010a) in claiming that the data point to the following generalisation.[2]

Generalisation: *superlative modified numerals are anti-specific*

[2]The terminology used in this chapter, however, differs completely from that in Nouwen 2010a, where I for instance do not use the notion of anti-specificity.

In section 3, I will give more substance to this characterisation. For now, it is exclusively meant as a descriptive generalisation: an expression like *at least n* cannot be used to refer to some specific number m. One manifestation of this is an epistemic effect. When a speaker uses *at least n* in a (simple) sentence, s/he cannot have a specific quantity in mind. One way of showing this is as in (9) (Nouwen 2010a). Whilst the speaker can reveal his or her knowledge of an exact number after describing that number imprecisely using a comparatively modified numeral, s/he cannot do so following a similar utterance containing a superlative numeral.

(9) a. John found more than 50 mistakes in the manuscript. 62, to be precise.
 b. John found at least 50 mistakes in the manuscript. #62, to be precise.

A second manifestation of the anti-specificity of *at least* and *at most* is non-epistemic. Generic statements with *at least* give rise to what I will call *variation readings*. For instance, (10) expresses that some of the computers sold have 2GB of memory, some have more, but none have less. The computers vary in the number of memory they have (which renders the contribution of *at least* anti-specific), but there is a lower bound of 2GB.[3]

(10) The computers we sell have at least 2GB of memory.

If we now turn to sentences where variation would not make any sense given our world knowledge, then the use of the superlatively modified numeral becomes unacceptable. For instance, given the fact that we know that octagons have a fixed number of sides (even though we might have forgotten which number that is), the variation reading enforced by anti-specificity on (11) renders the sentence infelicitous.[4]

(11) #An octagon has at most 10 sides.

This is clearly in sharp contrast to (12).

(12) An octagon has fewer than 10 sides.

Another way of stating the anti-specificity requirement is that superlative modified numerals require a variation of quantitative values. This variation has to be relative to some operator, such as for instance the generic quantification in (10). This example lacks an epistemic effect due to the fact that the generic allows for variation of the amount of memory over the set of computers. The

[3]There is an additional epistemic reading, where all computers sold have the same amount of memory, but the speaker is ignorant with respect to exactly what that amount is. Compare to footnote 4 and the discussion of (14) and (15) below.

[4]For some informants this sentence is acceptable with a epistemic (speaker ignorance) reading, which could be paraphrased as: *I can't remember how many sides an octagon has. It could be 10, or 9, or fewer, but I'm sure it is not more than 10.* It should be clear that this, too, is an anti-specific reading.

singular counterpart of (10) in (13) lacks such an operator. In that case an epistemic effect arises: the speaker is ignorant of the exact size of his or her computer's memory. It seems then that in the absence of an operator, a sentence with a superlative modified numeral is interpreted with respect to a covert epistemic operator (or perhaps a more pragmatic form of modality connected to the maxim of quality).

(13) My computer has at least 2GB of memory.

Variation also occurs under nominal and modal quantifiers. For instance, (14) has two readings. The least preferred is one where everyone found the same number of mistakes and the speaker does not know what that number is, but is aware that the number is not lower than 10. On the second, more preferred reading, there is variation among the number of mistakes everyone found, whilst no-one found fewer than 10.

(14) Everyone found at least 10 mistakes.

Exactly the same observation can be made with respect to modals.[5]

(15) To get tenure, John has to publish at least 3 books.

The epistemic reading concerns uncertainty about where the lower bound is. The speaker knows that once John has published a certain number of books (or more), he is eligible for tenure, but s/he is ignorant about the exact number in question. The variation reading is one in which the worlds in which John is eligible for tenure vary with respect to the number of books he has published: in each world this number is 3 or more; in none is it fewer than 3. In an example like (15) this reading resembles a free choice reading: once John is passed the 3 published books stage, he is eligible. That is, *any* number above 2 will do. On the other hand, an example like (15) is not ideal for testing whether the phenomenon in question is really like universal free choice. This is because for (15) it is rather unlikely that there is some further restriction on getting tenure, like not being allowed to have published 5 books, or more than 6. It turns out that modal variation reading is actually weaker than a proper universal

[5]A note on terminology is in order. In the works of Büring 2008 and Geurts and Nouwen 2007, the epistemic and variation readings are also distinguished, albeit only for modal sentences like (15). They receive different names from what I am using here, however. In Büring 2008, two kinds of readings are distinguished for examples like (15). What Büring calls the *authoritative reading* is what I call here the *variation reading*. This reading is authoritative in the sense that it is non-epistemic. The speaker is reporting on some specific belief s/he has about some quantity. In the case of (15) this is that the lower limit for tenure is publishing 3 books. What I have called the *epistemic reading* is called the *speaker insecurity reading* by Büring. Geurts and Nouwen propose that the difference between epistemic and variation readings is due to the modal nature of superlative modified numerals, where epistemic readings are straightforward compositional readings and variation readings are due to the fusion (so-called *modal concord*) of the modal inherent in the modified numeral with the explicit modal operator in the sentence. For that reason, Geurts and Nouwen call the latter kind of readings *modal concord* readings.

free choice reading.[6] We can see this by turning to an example like (17) in a situation like (16).

(16) *Password policy:* For security reasons, the system will not accept passwords that are shorter than 6 characters. Moreover, it cannot handle passwords that are longer than 10 characters.

(17) Passwords have to be at least 6 characters long.

On its modal variation reading, (17) is true in the context in (16). This reading cannot be a true (universal) free choice readings, since that would entail that any number of characters exceeding 5 would do for a password.

What the data point out is that superlative modified numerals are always anti-specific. That is, their essential use is to report variation with respect to some operator, relative to some bound. The operator in question can either be associated to an operator in the sentence (a plural/generic, a quantifier, a modal operator, etc.) or be implicitly derived from what the speaker considers possible, in which case an epistemic reading surfaces. The anti-specificity generalisation is descriptively accurate, but what is not clear from the data is what is responsible for it. In the next section, I will explore an account which likens anti-specificity to certain properties familiar from the literature on disjunction and implicature.

2 Anti-specificity by implicature

Superlative modified numerals do not just bear a resemblance to (some) epistemic indefinites, but also more generally to disjunction. The main piece of data is familiar from the *free choice* literature (Kamp 1973): simple sentences with disjunction carry an ignorance implicature (resulting in what I called an *epistemic reading*), while disjunctions embedded under a modal allow for both an ignorance and a free choice interpretation. The latter is akin what I have called the *variation reading*.[7]

[6] Thanks are due to Anastasia Giannakidou for pointing this out to me. This is furthermore reminiscent of Alonso-Ovalle and Menéndez-Benito's observation that the Spanish epistemic indefinite *algún* differs from, say, German *irgendein* in having a variation effect that is weaker than free choice (Alonso-Ovalle and Menéndez-Benito 2010). But see Lauer (2012) for a recent suggestion that the free choice effect of *irgendein* is in fact equally weak.

[7] In fact, the parallel between disjunction and class B numerals is stronger than that between epistemic indefinites and class B numerals. This becomes apparent by using the *Guess who?* test of Aloni and Port. Going back to the examples with indefinites I presented early on in this chapter, a speaker can remove an ignorance inference triggered by use of English *a* by continuing an example like, for instance, (5) with *Guess who?*, thereby indicating that s/he is aware of the identity of the referent in question. The same is not possible with German *irgendein*: the ignorance inference is not defeasible. Disjunction and class B numerals have ignorance effects that are strong, but slightly weaker than that of *irgendein*. For instance, (i) and (ii) are not that bad.

(i) John found at least 50 mistakes in the manuscript. Guess how many exactly?

(ii) John ate an apple or a pear. Guess which?

(18) John ate an apple or a pear, #namely a pear.

(19) John found at least 50 mistakes in the manuscript, #namely 62.

(20) John may eat an apple or a pear.
  a. *John may eat an apple* and *John may eat a pear.* — variation reading
  b. *John may eat an apple* or *John may eat a pear, but I don't know which.* — epistemic reading

(21) John's paper may be at most 10 pages long.
  a. *John's paper may be 10 pages long* and *it may be shorter than 10 pages* and *it may not be longer* — variation reading
  b. *John's paper may (only) be 10 pages long* or *it may (only) be 9 pages long* or *...etc. I don't know which it is.* — epistemic reading

The parallel goes beyond modals. As we saw in the previous section, superlative modified numerals create variation readings under nominal quantifiers and generics. That is, apart from an epistemic reading, examples like (22-a) and (22-b) have readings in which the number of mistakes varies with respect to the students, resp. the amount of memory varies with respect to the laptops. Similarly, free choice readings with disjunction are not limited to modals, but also occur with nominal quantifiers and generics, as in (23) (cf. Fox 2006; Nickel 2011).

(22) a. Every student found at least 10 mistakes. — variation/epistemic
  b. Our laptops come with at least 2GB of memory. — variation/epistemic

(23) a. Everyone ate an apple or a pear — variation/epistemic
  b. Our laptops come with a free mouse or a free insurance — variation/epistemic

On the basis of the parallel to disjunction, it has been proposed to attribute the effects that set superlatively modified numerals apart from their comparative counterparts to whatever mechanism is responsible for ignorance and free choice effects with disjunction (Nilsen 2007; Büring 2008; Cummins and Katsos 2010). No fully worked out analysis along these lines exists, however. One issue is that such studies focus only on *at least* and leave *at most* unanalysed (a fact to which I will return at the end of this work). Another is that these works lack a specific analysis of in what sense superlative modified numerals *are* disjunctions. An exception to this are the works of Schwarz (2012) and Mayr (2012), to which I will return to later too.

Given the lack of clarity of what it would essentially mean to draw a parallel between *at least* and disjunction, I will try to sketch the theoretical landscape

I am not entirely sure what the *guess who* test does and how it differs from, say, the test in (18) and (19).

as accurately as possible by exploring several options such analyses could take.

The driving intuition behind this line of approach is to think of class B quantifiers at some level as disjunctions. This may be thought especially appealing if we think of *at least* and *at most* as expressing a comparative relation that is inclusive of the numeral in question. That is, the idea is that superlative modifiers express the non-strict comparison relations '$\leq$' and '$\geq$', which, again on an intuitive level, corresponds to disjunction:

(24) non-strict comparison is a disjunctive relation

a. $x \leq n \quad := \quad x = n \vee x < n$

b. $x \geq n \quad := \quad x = n \vee x > n$

From here on, the analysis of disjunction and class B modified numerals are the same. Let me try to work things out following a fairly standard implicature mechanism (e.g. Geurts 2011; Grice 1978; Sauerland 2004). I will represent (informativity / Horn) scales as follows: $\langle \alpha, \beta \rangle$ expresses that α is an alternative to β and that it is strictly stronger than β. If there are two scales $\langle \alpha, \gamma \rangle$ and $\langle \beta, \gamma \rangle$, with α and β equally strong, I will write:

$$\left\langle \begin{array}{cc} \alpha & \\ & \gamma \\ \beta & \end{array} \right\rangle$$

This notation comes in handy for the case of disjunction, where both disjuncts are stronger than the disjunction itself. That is, $p \models p \vee q$ and $q \models p \vee q$. And so:

(25) a. $\left\langle \begin{array}{cc} p & \\ & p \vee q \\ q & \end{array} \right\rangle$

b. $\left\langle \begin{array}{cc} \text{John ate an apple} & \\ & \text{John ate an apple or a pear} \\ \text{John ate a pear} & \end{array} \right\rangle$

We can now follow the standard recipe for calculating scalar implicatures (see e.g. Sauerland 2004; Fox 2006; Geurts 2012): a weak utterance implicates that the speaker does not have the belief that any of the stronger alternatives are true. For disjunction, we get that an utterance of $p \vee q$ implicates both (26-a) and (26-b), where B is the speaker's belief operator.

(26) a. $\neg Bp$

b. $\neg Bq$

Combined, (26-a) and (26-b) give rise to an ignorance effect. The speaker lacks both the belief that p is true and the belief that q is true. Given that s/he asserted that $p \vee q$, she must believe that one of them is true, but as (26) indicates, s/he does not know which.

Compare this to an utterance containing *some*, given a Horn scale ⟨*every, some*⟩. There is now just one stronger alternative, and so we get a single implicature of the form in (27).

(27) $\neg B \forall x[p]$

The implicatures in (26) an (27) are called *weak* or *primary* implicatures. They can be strengthened once it is assumed that the speaker is knowledgable about the subject. Let us call this the authority assumption (Zimmermann 2000). Authority simply entails that for any proposition related to the subject matter the speaker either believes the proposition to be true or to be false:

(28) $B\varphi \vee B\neg\varphi$

Assuming this, (27) can be strenghtened to $B\neg\forall x[p]$, for if the speaker is knowledgable and s/he does not have the belief that p holds for every x, then it must be the case that s/he has the belief that p does not hold for every x.

Returning now to the case of disjunction, we could try to strengthen the two implicatures $\neg Bp$ and $\neg Bq$, on the authority assumptions that $Bp \vee B\neg p$ and $Bq \vee B\neg q$. This would give the stronger implicatures in (29): the speaker beliefs that both p and q are false.

(29) a. $B\neg p$
b. $B\neg q$

However, given that the speaker asserted $p \vee q$, we may assume that s/he believes $p \vee q$: $B(p \vee q)$. The problem with the strong implicatures in (29), however, is that they contradict this belief. It was therefore wrong of us to assume authority. This is how we can explain why disjunction only has the ignorance implicatures in (26). (Cf. Sauerland 2004; Geurts 2011.)

An exactly parallel reasoning applies to superlative modified numerals as soon as we assume that they are part of scales of a similar structure to those of disjunctions. For instance, given that $x \leq 20$ is entailed by both $x < 20$ and by $x = 20$, we get (30-a). In other words, superlative modified numerals are in a scale with both *exactly* numerals and comparative modified numerals, as in (30-b).

(30) a. $\left\langle \begin{array}{c} x < 20 \\ x = 20 \end{array} \quad x \leq 20 \right\rangle$
b. $\left\langle \begin{array}{l} \text{fewer than 20 A's B} \\ \text{exactly 20 A's B} \end{array} \quad \text{at most 20} \right\rangle$

Now, an utterance of the form $x \leq 20$ (say, *John found at most 20 mistakes in the manuscript*) will implicate (31-a) and (31-b).

(31) a. $\neg B(x < 20)$
b. $\neg B(x = 20)$

Given the speaker's utterance there is the belief $B(x \leq 20)$. The combination of the strengthened versions of (31), i.e. $B(x \geq 20) \wedge B(x \neq 20)$, are in direct contradiction to this assertion. As in the case of disjunction, only the weak implicatures arise. And, once more, these implicatures indicate ignorance: the speaker believes that x is in the 0 to 20 range, but s/he lacks any beliefs about where in that range x lies.

This illustrates the general mechanism for deriving epistemic effects with modified numerals: the effects are weak implicatures. Let us now turn to some complications that force us to be a bit more specific about what an approach along these lines would have to look like.

2.1 The relevant scales

The setup so far does not really have much to say about in what sense superlative modified numerals *are* disjunctions. Above I alluded to the general intuition that $x \leq 20$ may be considered as a disjunction of $x = 20 \vee x < 20$, and it is, in fact, often pronounced as such. However, any such comparative relation is equivalent to a disjunction, including strict comparisons like those worded by comparatively modified numerals: $x < 20$ equals $x = 19 \vee x > 19$.[8] If being equivalent to a disjunction is all that is needed to have a parallel scalar structure to disjunction, then we would expect the following scale for an expression like *fewer than 19*:

(32) $\left\langle \begin{array}{l} \text{fewer than 19} \\ \text{exactly 19} \end{array} \quad \text{fewer than 20} \right\rangle$

This way we come to expect ignorance effects for comparatively modified numerals too, for an utterance of the form $x < 20$ now implicates $\neg Bx < 19$ and $\neg Bx = 19$.[9]

The upshot is that we will need to stipulate that superlative modifiers are part of Horn scales that have a different structure from those of which comparatively modifiers are part. In particular, whilst the scales in (34) are informativity scales (i.e. they are in accordance with sentence-level entailments), it needs to be stipulated that these scales are not used for the calculation of implicatures.

(33) a. $\left\langle \begin{array}{l} \text{fewer than } n \\ \text{exactly } n \end{array} \quad \text{at most } n \right\rangle$

b. $\left\langle \begin{array}{l} \text{more than } n \\ \text{exactly } n \end{array} \quad \text{at least } n \right\rangle$

(34) a. $*\left\langle \begin{array}{l} \text{fewer than } n-1 \\ \text{exactly } n-1 \end{array} \quad \text{fewer than } n \right\rangle$

[8] Assuming x ranges over counts of discrete entities.

[9] I actually do not believe that this is a bad prediction, for there certainly are highly defeasible ignorance implicatures for *more than n*. The important point to make, however, is that what we need to account for is why the epistemic effects of *at least* are so much stronger and so much more inescapable. Assuming the same kind of scales for both comparative and superlative modifiers will not get us there.

b. $*\left\langle \begin{array}{l} \text{more than } n+1 \\ \text{exactly } n+1 \end{array} \quad \text{more than } n \right\rangle$

At this point, one might think that it suffices to stipulate that expressions denoting inclusive comparison relations ($\leq$ / $\geq$) are associated to the kind of branching Horn scales as in (33), but that expressions denoting strict comparison relations ($<$/$>$) are never associated to such scales.[10] This will not do for two reasons. First of all, there are a number of expressions that denote non-strict, i.e. inclusive, comparison but that lack the kind of ignorance inferences *at least* and *at most* do. Equatives are the first example I will give. It is commonly assumed (e.g. Klein 1980) that the equative expresses non-strict comparison. Such analyses allow to derive strong readings of equatives (those equivalent to *exactly as ... as ...*) by quantity implicature.

(35) John is as tall as Bill

a. *John's height* $\geq$ *Bill's height* — assertion
b. *John is not taller than Bill* — implicature

The implicature in (35-b) is based on a scale different in structure from those in (33), namely (36):

(36) $\langle$ taller, as tall as $\rangle$

Similarly, if it was the $\geq$ and $\leq$ relation that triggered the implicatures with *at least* and *at most*, then we come to expect that negated comparatively modified numerals give rise to the same implicatures, given that the negation of $>$ is $\leq$ and the negation of $<$ is $\geq$. As I observed in Nouwen (2008b), it turns out, however, that negated comparatives behave like equatives. That is (37) is typically understood as conveying that John found exactly 22 mistakes (with the additional inference that 22 is a lot).

(37) John found no fewer than 22 mistakes.

A similar point can be made by turning to Chinese data from Nouwen (2010a). Both (38) and (39) express that the number of sides in a triangle is 3 or more. In other words, both sentences involve an inclusive comparison relation. The two ways of expressing this same relation, $x \geq 2$, differ only in what we have called the anti-specificity effect. Whilst (38), in parallel to the English anti-specific modified numerals, is unacceptable, the phrasing in (39) of the same content is fine. Hence, Chinese has two ways of expressing $\geq$, one anti-specific, the other not.

(38) #Sanjiaoxing zui-shao you liang-tiao bian.
triangle most-little have 2-CL side
3≥2

[10]The approach of Cummins and Katsos 2010 is an example of a theory doing that.

(39) Sanjiaoxing zhi-shao you liang-tiao bian.
triangle to-little have 2-CL side
$3 \geq 2$

The conclusion is that for the implicature view on the epistemic nature of some modified numerals to work, we will need to stipulate, on a case by case basis, that the anti-specific modified numerals are exactly those that are associated with the kind of branching Horn scales that we also attribute to disjunction.

2.2 Modal variation readings

Whilst the scale stipulations needed to make the implicature view work are not the most elegant form of analysing the data, one may have the hope that ultimately we will find an independent phenomenon that accounts for why anti-specific modified numerals are part of the scales responsible for epistemic effects. In section 3 and 4, below, I will suggest a potential way of overcoming the need for stipulation. Let us for now, however, give the implicature view the benefit of the doubt and ignore the stipulative nature of the scales on which the epistemic effects are based. The next step is then to see how the implicature view deals with environments where the epistemic effect disappears. Consider (40).

(40) John's paper is required to be at least 20 pages long.

Ignoring the ignorance reading, the modal variation reading is straightforwardly derived as follows. (Compare to Sauerland (2004) and Fox (2006) for discussion of the parallel case involving disjunction.) Let us firstly assume that (40) is interpreted as (41).

(41) In all worlds in which John's paper meets the requirements, the number of pages ≥ 20.

Let us moreover abstract away from the content and represent this as $\Box[x \geq 20]$. The modal is monotone with respect to the scale structure (modals preserve entailments), so we can assume that at sentence-level, the relevant alternatives are as in (42).

(42) $$\left\langle \begin{array}{c} \Box[x > 20] \\ \Box[x = 20] \end{array} \quad \Box[x \geq 20] \right\rangle$$

Now, the implicatures for (41) are going to be $\neg B\Box[x > 20]$ and $\neg B\Box[x = 20]$. In combination with the belief behind the assertion itself and if we assume that the speaker is knowledgable, then the strengthened version of this gives us:

(43) $B\neg\Box[x > 20] \wedge B\neg\Box[x = 20] \wedge B\Box[x \geq 20]$

Crucially, (43) is not contradictory. So, in contrast to the non-modal sentences, modal ones are going to give rise to strong implicatures. Furthermore, (43) exactly characterises the interpretation we want: according to the speaker, John's paper cannot be shorter than 20 pages (3rd conjunct) but it does not matter if it is exactly 20 pages or longer (first 2 conjuncts). (See Schwarz (2012) for a more advanced discussion along a similar route.)

2.3 Interim conclusion

The implicature view successfully accounts for epistemic and modal variation effects of *at least*, on the basis that for the sake of implicature calculation, superlative quantifiers are exactly like disjunctions. The drawback of this account is that it is unclear on what basis superlatives and disjunctions are alike. It would be desirable to find a way of explaining why the scales associated with anti-specific modified numerals are exactly the ones we need to derive the right implicatures. What I will show in the remainder of the paper is that this shortcoming can be resolved once we look at the full class of modified numerals that trigger epistemic and variation effects. This I will do in the next section. After that, I will show that, by comparison to an analysis from the epistemic indefinites literature, the insights we gain from from looking beyond superlative modified numerals provide a different, better motivated, perspective on the implicature approach I have just presented.

3 Anti-specificity by presupposition

So far, we have only discussed the contrast between comparative and superlative modified numerals. As I argued in Nouwen (2010a), this contrast is part of a larger phenomenon. That is, many languages distinguish two kinds of modified numerals, those that fall under the anti-specificity generalisation discussed in section 1 and those that do not. In that article, I called the latter group, among which comparative modified numerals, *class A*, and the former one, which includes superlative modified numerals, *class B*. I will use the same terminology in what follows.

In English, table 1 summarises the data. The modified numerals on the right of the table share with *at least* and *at most* the alternation between an epistemic and a variation effect, whilst the expressions on the left of the table share with *more than* and *fewer/less than* the lack of such effects.

So, for instance, it is unacceptable to say that a triangle has *maximally 10 sides*, and if I tell you that there are *maximally 60 people* in my class, I suggest that I do not know the exact number, or, alternatively, I may be suggesting that the number varies. (See Nouwen 2010a for more examples.)

There is some suggestive evidence that, cross-linguistically, there is remarkably little variation in what a table like this looks like. (See, again, Nouwen (2010a) for some tentative observations.) In language after language that I looked at, the comparative modified numerals fall in class A, whilst the superlative ones are class B. Furthermore, prepositional modified numerals appear to be systematically split according to the locative/directional distinction. That is, prepositions that express the specific location of an object, like *over* or *under* in

English

A	B
more than n	at least n
less/fewer than n	at most n
over n	up to n
under n	from n
between n and m	from n to m
	minimally n
	maximally n
	n or more
	n or fewer

Table 1: The class A/B distinction in English

English, are class A, whilst directional (or *dynamic*) prepositions, which relate to a path being traversed (*from*, *up to*), are class B. This suggests that we should look for something that (adverbial) superlatives, directional prepositions, disjunctions and adverbs like *maximally* have in common. The hypothesis is now that the semantic mechanisms behind all class B expressions are all mechanisms that enforce anti-specificity, not just in the combination with a numeral, but in their general use.[11]

(44) **Class B expressions are expressions that (happen to) presuppose anti-specificity**

Support for this comes from what we know about the expressions that form class B modifiers. Let me start with superlatives. Consider (45):

(45) #The tallest queen of the Netherlands is called Beatrix.

To most speakers, (45) is unacceptable. The reason is that there is just one queen of the Netherlands, which renders the qualification *tallest* superfluous. Beatrix is not just the tallest queen of the Netherlands, she is also the shortest: superlativity only makes sense if (the degree properties of) multiple individuals are compared. For some people, (45) may have an interpretation, but this will be one in which the sentence states that out of all queens the Netherlands has ever had, Beatrix was the one who was tallest. Note the relation to anti-specificity:

[11] I am treating anti-specificity here as a presupposition. An anonymous reviewer rightly complains that I do not discuss how class B expressions display the tell-tale projection signs of presupposition triggers. The problem here is that class B expressions resist embedding, especially under negation. As the reviewer comments, it may be more appropriate to refer to anti-specificity as a lexical requirement, rather than a presupposition. I can see that this would be more neutral terminology, but am not really sure that in the end this would be so much different from calling it a presupposition and so, in line with work on superlativity referred to below, I will keep on doing so. Keep in mind that the main point is that anti-specificity is a property that is lexically associated to class B expressions.

(45) is unacceptable because of there being just a single (current) queen, but can (for some) be saved by interpreting the domain of the superlative not as a singleton, but as a range of queens. The same observation can be made on the basis of (46).

(46) The highest amount of prize money that Federer won so far is $1 million.

A speaker of (46) could mean many things, for instance that the highest prize money Federer won for an edition of Wimbledon is $1 million, or that the most he has won in a year is $1 million. As long as there is some way of carving up Federer's earnings beyond lumping it all in one singleton set: (46) cannot mean that Federer won $1 million throughout his whole career.

In the literature on superlativity, this anti-specificity effect has been recognised, especially via the influential manuscript Heim 1999 (cf. Hackl 2009; Gajewski 2012 for some recent discussion.) Heim proposes that any superlative comes with three presuppositions. For instance, (47) presupposes (48), and expresses that (49) is the case.

(47) Tim is the loudest Canadian John knows.

(48)
  a. There is a comparison class C of individuals, one of which is Tim.
  b. Every individual in C is a Canadian John knows who moreover has a degree of loudness
  c. The class C is not a singleton

(49) Every individual in C that is not Tim is less loud than Tim.

In summary, there are reasons to assume that anti-specificity is a property of superlativity in general, not just of superlative numeral modifiers. A similar reasoning applies to the case of directional prepositions (cf. Nouwen 2008a). Unlike locative prepositions, which refer to the absolute location of their subject, directional prepositions express locations relative to some path. This is why (50-a) is acceptable, but (50-b) is not.

(50)
  a. John ran up to here.
  b. #John is standing up to here.

Similarly, in the temporal domain, *before* behaves like a locative, whilst *until* behaves like a directional.

(51)
  a. John worked until midnight.
  b. #John arrived until midnight.
  c. John arrived before midnight.

Expressions like *up to* and *until* appear to have a requirement that once more resembles anti-specificity: they are incompatible with single values and instead require a non-singleton set, such as a path or an interval. (See Bennett and Partee 1972; Pinon 1993 for worked out analyses that may be interpreted as imposing such a requirement.) Importantly, not only have such expressions a non-singleton requirement, they moreover show the variability effects we have discussed above. That is, the unacceptability of (50-b) and (51-b) is obviated in the scope of a modal.

(52) a. John is allowed to stand up to here.
b. John is allowed to arrive until midnight.

The proposal can be summarised as follows: class B modifiers refer to end-points, that is, maxima or minima. Only non-singletons, that is, only *ranges* (or *paths*, *intervals*, etc.), have end-points. As such, they presuppose that their domain is not a singleton.

The next step in the analysis is to see what the relation is between anti-specificity requirements and the data of section 1. In Nouwen 2010a and Penka 2010, it is proposed that the epistemic and variation effects are due to the fact that whenever a modified numeral takes scope over an operator, such as a modal or some other kind of quantifier, the domain of the modified numeral is not a singleton. In other words, taking scope over an operator satisfies the anti-specificity presupposition of class B modified numerals. Whilst this approach is very successful as an analysis of *at most* and other upper bound class B numerals, it runs into serious problems with lower bound quantifiers. (See Nouwen 2010a and Nouwen 2010b for detailed discussion of the complications.) Importantly, the anti-specificity presupposition is fully independent of the mechanisms used to derive epistemic and modal variation readings. In the next section, I will explore how such a presupposition fares when paired with an implicature-based analysis of the various readings. The goal will be to at least replicate the results of the account presented in section 2, but now with a better motivated basis.

4 Combining presupposition and implicature

The characterisation presented in the previous section is reminiscent to the discussion of Alonso-Ovalle and Menéndez-Benito 2010 of the Spanish epistemic indefinite *algún*, which claims that *algún* imposes an anti-singleton constraint on the domain of quantification. The mechanism that accounts for the modal variation and epistemic effect readings in that work, moreover, more closely resembles the implicature account I presented in section 2. Let me illustrate with an example.

The sentence in (53) means that the speaker is sure that Juan is in some room in the house. S/he may know of some rooms that Juan definitely is not in, but it can't be that s/he knows which room he is in: there have to be at least two rooms in the domain that are possibilities (Alonso-Ovalle and Menéndez-Benito 2010).

(53) Juan tiene estar en alguna habitación de la case.
Juan has to be in ALGUNA room of the house.
'Juan has to be in a room of the house.'

The analysis offered by Alonso-Ovalle and Menéndez-Benito is that *algún* signals that the domain of quantification is not a singleton. That is, (53) asserts (54-a) and presupposes (54-b):

(54) a. Assertion: $\Box\exists x[x \in f(\text{room}) \wedge \text{in}(j, x)]$
b. Presupposition: $|f(\text{room})| > 1$

Here, f selects the domain of quantification for the existential quantifier: $f(\text{room})$ denotes some subset of the set of rooms. The presupposition says that this subset consists of more than one room.

The configuration in (54) now triggers implicatures. The idea is that these are based on reasoning why the speaker has used an expression which comes with an anti-specificity presupposition. Based on a proposal in Kratzer and Shimoyama 2002, Alonso-Ovalle and Menéndez-Benito propose that the configuration in (54) raises the issue why the speaker did not use a smaller domain, thereby making a stronger statement. In particular, then, the implicatures arise that no singleton domain would make (54-a) true and so that:

(55) a. $\neg\Box\exists x[x \in \{\text{bedroom}\} \wedge \text{in}(j, x)]$
b. $\neg\Box\exists x[x \in \{\text{living-room}\} \wedge \text{in}(j, x)]$
c. $\neg\Box\exists x[x \in \{\text{kitchen}\} \wedge \text{in}(j, x)]$
d. etc.

Together, (54) and (55) accurately represent the modal variability effect of *algún*. Note the parallel to the implicature mechanism presented in section 2. Assuming that a set of rooms like {bedroom, living-room, kitchen}, the relevant scale is:

$$\left\langle \begin{array}{l} \Box in(j, \text{bedroom}) \\ \Box in(j, \text{living-room}) \\ \Box in(j, \text{kitchen}) \end{array}, \quad \Box\exists x[x \in f(\text{room}) \wedge \text{in}(j, x)] \right\rangle$$

In contrast to the analysis I presented in section 2, however, the motivation for this scale is directly linked to the presupposition that comes with *algún*. By presupposing that the relevant domain is non-singleton, the reason why the speaker did not assert any of the (stronger) alternatives where the domain *is* a singleton become relevant. As in section 2, a hearer can conclude that such stronger alternatives are false. As such, the presuppositions we have identified in section 3, can help motivate an implicature analysis of modal variation as the one in 2. Before we turn to an application of this idea to modified numerals, let us now turn to epistemic readings of *algún*.

Alonso-Ovalle and Menéndez-Benito attribute the epistemic effect of the epistemic indefinite *algún* to a covert modal. They assume assertions involve universal quantification over speaker-related doxastic alternatives. So, a non-modal version of (53) receives exactly the same analysis as we find in (54)/(55).

(56) Juan está en alguna habitación de la case.
Juan is in ALGUNA room of the house.
'Juan is in a room in the house (and I don't know which; there are several possibilities left open)'

I suppose it is safe to assume that what Alonso-Ovalle and Menéndez-Benito have in mind comes very close to the use of the belief operator B for the implicatures that come with disjunction, as presented in section 2. In other words, for (56), we get:

(57) a. Assertion: $B\exists x[x \in f(\text{room}) \wedge \text{in}(j, x)]$
b. Presupposition: $|f(\text{room})| > 1$
c. Implicature: $\neg\exists y[\text{room}(y) \wedge B\exists x[x \in f(\{y\}) \wedge \text{in}(j, x)]]$
$= \neg\exists x[\text{room}(x) \wedge B\text{in}(j, x)]$

The hope is now that given the parallel between epistemic indefinites and class B modified numerals, the above analysis of *algún* may extend to expressions like *at least* and its kin, too. Let us assume that lower-bound class B expressions like *at least 2* and *minimally 2* express that a certain set has a cardinality that is or exceeds 2 (as I did before). Let us furthermore assume that the domain for this *cardinality* is restricted in the same way as the domain of regular indefinites is restricted. Class B modified numerals furthermore presuppose that the restricted domain is not a singleton. Formally, this means that we interpret an example like (58) as in (59).

(58) John needs to write at least 2 pages.

(59) a. Assertion: $\Box[|\lambda x.\text{page}(x) \wedge \text{write}(j, x)| \in f(\{n : n \geq 2\})]$
b. Presupposition: $|f(\{n : n \geq 2\})| > 1$

Note first of all that the assertion entails that John cannot write fewer than 2 pages. This is as it should be, since (58) expresses a lower bound on the number of pages John can write. The configuration in (59) once more generates implicatures. By using a superlative modifier, the speaker presupposes (59-b). This raises the issue why a singleton domain is excluded by the speaker, i.e. why the speaker cannot narrow the cardinality in question down to a single option. S/he thereby implicates that there is no f that assigns a singleton to $\{n : n \geq 2\}$ such that (59-a) is (thought to be) true (by the speaker). The result is a series of implicatures: for each n exceeding 1 there is no requirement that John writes exactly n books. The implicatures of (59) are then:

(60) It is not necessary that John writes exactly 2 pages
It is not necessary that John writes exactly 3 pages
It is not necessary that John writes exactly 4 pages
etc.

This gives us a modal variation effect, but one that is not yet entirely equivalent to what we would derive in the framework set up in section 2.[12] The contrast is summarised in (61) and (62).

(61) Anti-specific approach to $\Box[x \geq 2]$

[12] I am indebted to Luis Alonso-Ovalle and Paula Menéndez-Benito for pointing out that I missed this difference in an earlier version of the manuscript.

a. Alternatives: $\langle \ldots, \ \Box[x=3], \ \Box[x=2], \ \Box[x \geq 2]\rangle$
b. Implicatures: $\neg\exists n \geq 2 \Box[x=n]$

(62) Disjunction approach to $\Box[x \geq 2]$

a. Alternatives: $\left\langle \begin{array}{cc} \Box[x=2] & \\ & \Box[x \geq 2] \\ \Box[x>2] & \end{array} \right\rangle$
b. Implicatures: $\neg\Box[x=2] \wedge \neg\Box[x>2]$

From the implicatures in the disjunction approach, (62-b), it follows that $\Diamond[x=2]$, that John is allowed to write (just) three pages. This appears to be a desirable prediction for examples like (58), for (58) seems to suggest that John will meet the requirements as soon as he writes three pages. But the same does not yet follow from (61-b): the implicatures triggered by the anti-specific approach are compatible with the lower bound exceeding two. (That is, they are compatible with $\Box\neg(x=2)$.) There are some cases, however, that suggest that the set of implicatures triggered by anti-specificity in (61-b) is more appropriate than those triggered by a disjunction scale. Take the following case, suggested to me by Florian Schwarz (p.c.).

(63) In line with safety regulations, all our buildings have at least 2 stairways. In reality, however, they have even more.

Following a strategy that parallels (59), the first sentence in (63) asserts that for all buildings the number of staircases is in some subset of $\{2, 3, 4, \ldots\}$ whilst presupposing that that subset is not a singleton. The triggered implicatures amount to saying that for no specific n is it the case that all building have exactly n staircases. This is compatible with a situation described by the second sentence in (63), say one in which all buildings have between 3 and 5 stairways.

However, our intuitions for examples like (58) suggest that for some (perhaps many) cases, we somehow need to derive additional implicatures on top of those predicted by the anti-specificity approach. That is, to get the inference that $\Diamond[x=3]$, we do not need to assume a disjunctive scale, but we could instead turn to additional alternatives on top of those motivated by anti-specificity. The following picture emerges then:

(64) a. Alternatives: $\left\langle \begin{array}{cccc} \ldots \ \Box[x=4] & \Box[x=3] & \Box[x=2] & \\ & & & \Box[x \geq 2] \\ \ldots \ \Box[x \geq 5] & \Box[x \geq 4] & \Box[x \geq 3] & \end{array} \right\rangle$
b. Implicatures: $\neg\exists n \geq 2 \Box[x=n] \wedge \neg\exists n > 2 \Box[x \geq n]$

From the implicatures in (64), together with the assertion $\Box[x \geq 2]$, it follows that $\Diamond[x=2]$. What the discussion so far illustrates is that we can motivate part of the alternatives in this option (those of the form $x=n$) by alluding to the lexical property of anti-specificity presuppositions. The remaining alternatives – of the form $x \geq n$ – are based on the numeral scale itself. Examples such as (63) suggest that the implicatures triggered by these alternatives may perhaps be more defeasible than those associated to the anti-specificity presupposition.

Without a more precise understanding of the data, however, this remains a rather speculative statement.

Interestingly, Schwarz (2012) explores the option summarised in (64) in great detail (as well as other, related options). He observes (in parallel to a similar observation by Mayr 2012) that such rich scales beg the question why part of the implicatures in (64-b) do not end up being strengthened. For instance, a strong (i.e. secondary) implicature $\Box\neg(x = 3)$ or $\Box\neg(x \geq 4)$ does not contradict other primary implicatures. Such strengthened implicatures are obviously, however, unwanted. Schwarz concludes that an account along these lines will have to come with a mechanism that guards consistency during the derivation of implicatures that is more advanced than that of Sauerland (2004).

In any case, an approach along the lines of (63) would still need to provide independent motivation for the additional set of alternatives of the form $x \geq n$, and gain more clarity of what the data reveal in detail regarding free choice inferences. In this respect, the epistemic reading of, say, (65-a) once more provides a quite clear intuition that the implicatures due to anti-specificity are often insufficient: (65-a) appears to suggest that the speaker believes it possible that John wrote 2 pages.

To account for the epistemic effect, we could as in Alonso-Ovalle and Menéndez-Benito's proposal for *algún* assume that assertions contain a universal doxastic modal, which creates the same configuration as in (59).

(65)
a. John wrote at least 2 pages.
b. Assertion: $B[|\lambda x.\text{page}(x) \land \text{wrote}(j, x)| \in f(\{n : n \geq 2\})]$
c. Presupposition: $|f(\{n : n \geq 2\})| > 1$

Thus, (65-a) raises the issue why the speaker did not make a stronger assertion based on a narrowed domain. This implicates that there is no $n \geq 2$ such that the speaker *believes* that John has written exactly n pages. On top of this, we will have to assume additional implicatures that excludes that the speaker believes that the number of pages John wrote is 3 or more. That is, the analysis of the epistemic reading would have to be completely parallel to the suggested strategy in (64).

5 Discussion

I have shown that the results of an implicature-based account of *at least*, such as for instance that of Büring (2008), can be replicated by generating implicatures on the basis of the numeral modifier imposing an anti-specificity presupposition. The advantage of the approach outlined in the previous section is that there is no need to stipulate the alternatives that come with superlative modifiers, and that there is no need to claim that class B quantifiers are, in some ill-understood sense, disjunctive. Unfortunately, such a step does necessitate the assumption of further alternatives, which lack a similar motivation. This drawback may not seem unsurmountable. However, the strategy unfolded above has one quite dramatic further issue: it fails for upper bound modified numerals, in particular in combination with existential modality. In fact, this is a drawback it inherits from the account in section 2. To see this, let us consider the variation reading of (66).

(66) John is allowed to write at most 20 pages.

This example has a clear non-epistemic reading, namely one where it states that the (upper) page limit is 20. At sentence-level, the relevant scale is (67) (once more, the monotonicity of the modal preserves the scale structure associated to *at most*):

(67) $$\left\langle \begin{array}{c} \Diamond[x < 20] \\ \Diamond[x = 20] \end{array} \quad \Diamond[x \leq 20] \right\rangle$$

Uttering (66) will implicate that $\neg B\Diamond[x < 20]$ and $\neg B\Diamond[x = 20]$, which together with the assertion yields (68). Strengthening will lead to the beliefs in (69).

(68) $\neg B\Diamond[x < 20] \wedge \neg B\Diamond[x = 20] \wedge B\Diamond[x \leq 20]$

(69) $B\neg\Diamond[x < 20] \wedge B\neg\Diamond[x = 20] \wedge B\Diamond[x \leq 20]$

The interpretation in (69) is clearly inconsistent. It says that the speaker has the belief that there is a possibility that x is in the 0-20 range, whilst at the same time s/he has the beliefs that x cannot be 20 and that x cannot be smaller than 20. The interpretation based on weak implicatures in (68) is not inconsistent. One can have the belief that it is possible that x is in the 0-20 range without in particular believing that it is possible that x is in the 0-19 range and without believing that it is possible that x equals 20. Importantly, however, this is not one of the salient readings we observe for (66). As an interpretation for (66), (68) is obviously much too weak: it merely states that it is fine for John's paper to be in the 0-20 page range without saying anything about whether or not a longer paper is allowed. The modal variation reading, on the other hand, expresses an upper bound: it entails $\neg\Diamond[x > 20]$.

Note that even if we change the scale into a simple $\langle\Diamond[x \leq 19], \Diamond[x \leq 20]\rangle$, we get the wrong result, since such a scale would result in the implicature that John's paper is not allowed to be *shorter* than 20 pages, while our goal is to derive the implicature that the paper is prohibited from being *longer* than that.

Similar issues arise when we have implicatures triggered by reasoning about an anti-specificity presupposition. Following the recipe I outlined in the previous section, we get the following interpretation for (66).

(70) a. Assertion: $\Diamond[|\lambda x.\text{pages}(x) \wedge \text{write}(j, x)| \in f(\{n : n \leq 20\})]$
b. Presupposition: $|f(\{n : n \leq 20\})| > 1$

The problem is that the assertion is too weak for the modal variation reading. The formula in (70-a) says that it is allowed for John to write a number of pages that does not exceed 20, but it says nothing about whether or not he is allowed to write more. That is, (70-a) fails to capture the upper limit expressed by (66).

Implicatures based on reasons why the speaker did not use a smaller domain are not going to help here. We would derive the implicature that for no $n \leq 20$ is it allowed that John writes exactly n pages. This contradicts the assertion.

Alonso-Ovalle and Menéndez-Benito have a similar problem and use a different set of implicatures, based on Kratzer and Shimoyama (2002), to account for modal variation effects involving existential modals. The idea is that a speaker may not just widen the domain to avoid making a false claim, but also because the smaller domains may wrongly be interpreted exhaustively. That is, if $f(n : n \leq 20)$ were $\{18\}$, then an exhaustive interpretation suggests that John is not allowed to write any other number of pages. Widening the domain to a non-singleton, then implicates that no such exhaustive reading is available. Formally, the implicature is:

(71) $$\forall n \leq 20[(\Diamond[|\lambda x.\text{page}(x) \wedge \text{write}(j,x)| = n]) \rightarrow \\ \exists n' \leq 20[n' \neq n \wedge \Diamond[|\lambda x.\text{page}(x) \wedge \text{write}(j,x)| = n']]$$

This says that if it is true that John is allowed to write n pages, then there will be some other number of pages (≤ 20) that he is also allowed to write. There is no number such that this is the number he *has* to write. The implicature in (71) successfully accounts for the modal variation effect. However, it still does not create an upper bound. In other words, whilst the above comparison to Alonso-Ovalle and Menéndez-Benito's analysis of *algún* offers a route to a stipulation-free implicature account of class-B modified numerals, such an analysis fails on upper bound class B quantifiers. This is once more completely parallel to the disjunctive scale approach of section 2. In fact, the literature lacks a successful implicature-based account of *at most* and its kin, witness the focus on *at least* in works such as Büring (2008) and Cummins and Katsos (2010).

Despite this limited empirical coverage, the conclusion that I wish to draw from the above is that rather than a comparison to disjunction, it is more helpful to compare class B expressions to a much more general class of expressions that come with domain-related presuppositions. That is, the class B expressions are not just to be likened to disjunction, but also to free choice items, epistemic indefinites generally and more remote phenomena like superlativity, directional prepositions and durative adverbs. The anti-specificity requirement that is an integral part of such phenomena becomes part of the meaning of a modified numeral once such phenomena are applied to numerical quantification. The implicatures of class B expressions are then implicatures based on reasoning why the speaker has used a form that comes with such a requirement. In other words, the scalar alternatives for a sentence containing a modified numeral are (in part) based on the presuppositions triggered by the modifier in question.

The fact that the account I have proposed above runs into serious problems for *at most* results in a remarkable state of affairs. On the theoretical table, there are two proposals that have class B quantifiers presuppose anti-specificity, that of Nouwen (2010a) and that of this paper. The former theory deals effortlessly with *at most*, but fails to make the correct predictions for *at least* — exactly the opposite picture from the empirical coverage of the account I have pursued here. I see no easy reconciliation between the two approaches, so I will leave it for future research to study how to use anti-specificity for a more broadly successful approach to modified numerals.

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