Numeral Semantics | Thursday

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ESSLLI 2019

bit.ly/esslli-numsem

Imprecision

(1) It's three o'clock.

(2) Half a million people joined the protest march.

Imprecision

(1) It's three o'clock. $\rightsquigarrow$ exactly 3pm

(2) Half a million people joined the protest march.

$\rightsquigarrow$ at least / exactly 500,000 people joined

Imprecision

(1) It's three o'clock.

(2) Half a million people joined the protest march.

Intuition:

(1) is true (enough) at 2:59

(2) is true (enough) if 498,923 people joined

Imprecision

(1) It's three o'clock

You are in charge of a rocket launch. A computer has been programmed to check all safefy features in such a way that this safety check will be concluded at exactly 3pm, which will be the time of ignition. You are awaiting the results of the check and are in charge of giving the order to launch the rocket at this precise time. In this context, (1) is not true enough at 2:59.

Imprecision

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(2) Half a million people joined. true if 498,923 joined

(3) 498,923 people joined. false if 498,922 joined

Plan

- Imprecise semantics
- Numerals, roundness, granularity
- A theory of granularity
- Loose ends and connected thoughts

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- Numerals, roundness, granularity
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Caveat: today's lecture presumes we have a compositional interpretation for numerals

⟦Twelve students came to the party⟧

Imprecise semantics

Fifty students came to the party

true iff the number of students that came to the party is in the interval $[50 - \delta, 50 + \delta]$

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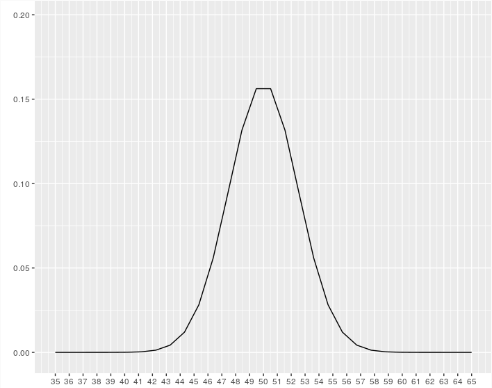

Probabilistic interpretation

$[\![\varphi]\!]^w$ = some truth-value

$[\![\varphi]\!] = \lambda w.[\![\varphi]\!]^w = 1$ = truth-conditions

Alternative: interpretation of φ yields a probability distribution over a space of possible worlds

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John read four Harry Potter novels

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truth-value	0	0	0	0	1	0	0	0

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truth-value	0	0	0	0	1	1	1	1
probability	0	0	0	0	.8	.15	.025	.025

Probabilistic interpretation

$[\![u]\!]^w$: a truth-value

$[\![u]\!] = \lambda w.[\![u]\!]^w = 1$: truth-conditions

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$P(w|u)$ the probability that w is the actual world, given utterance u

Probabilistic interpretation

$[\![u]\!]^w$: a truth-value

$[\![u]\!] = \lambda w.[\![u]\!]^w = 1$: truth-conditions

$P(w|u)$ the probability that w is the actual world, given utterance u

Recovering the truth-conditions of u:

$$\lambda w.P(w|u) > 0$$

Round numbers

- Observation: so-called round numbers tend to have imprecise meanings

I invited 50 people	I invited 49 people
This laptop costs 1000 euro	This laptop costs 987 euro
Add 250 grams of flour	Add 253 grams of flour

Round numbers

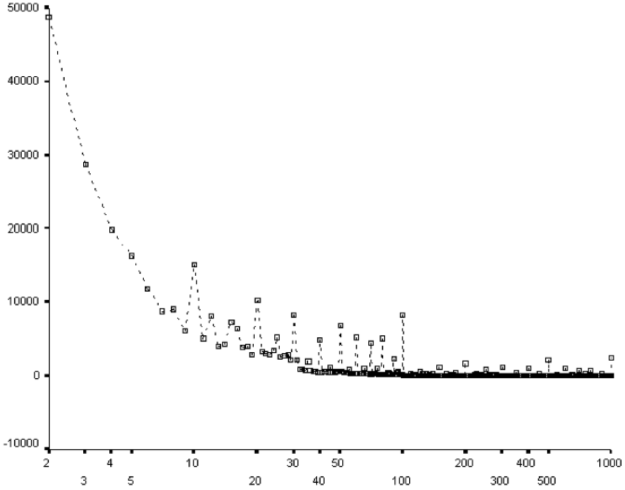

Fig. 1. Observed number frequencies.

Round numbers

Jansen & Pollmann:

- 2-ness
- 5-ness
- 10-ness
- 2,5-ness

- predict frequency in a corpus (nothing else does really)

Round numbers

Round numbers

Round numbers

- Roundness correlates with:
- frequency
- contextual salience
- morphological complexity

- From now on: we just assume we know what is round and not round in a given context

Roundness, precision and granularity

- Roundness is a factor in precision
- But not a decisive one

I invited 50 people

I invited 49 people

- Granularity

Granularity

A flat surface

  

Granularity

A flat surface

   

Granularity

He moved his hand to the door handle, grabbed it and moved it downwards. Then, while still holding the handle, he pulled the door towards himself.

Granularity

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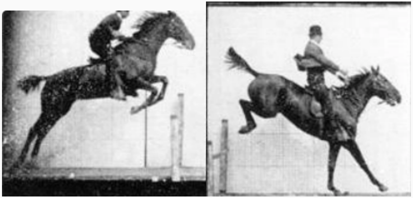
Compare to:

He opened the door.

Granularity

The horse jumped over the fence.



$$\exists t, t', x, y[horse(x) \wedge fence(y) \wedge t' > t \wedge leftof(x, y, t) \wedge leftof(y, x, t')]$$

Granularity

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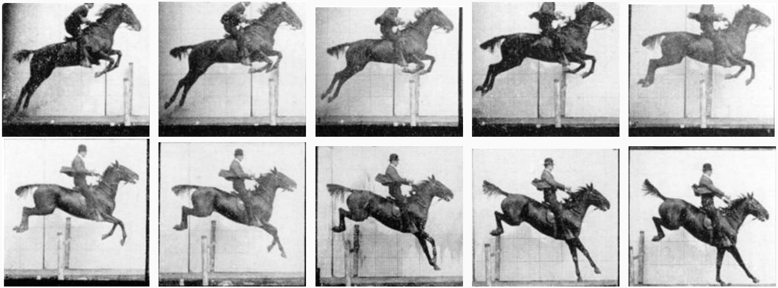

$\exists t_1 \ldots t_{10}, x, y[\mathit{horse}(x) \wedge \mathit{fence}(y) \wedge t_1 < t_2 < \ldots < t_{10} \wedge \ldots]$

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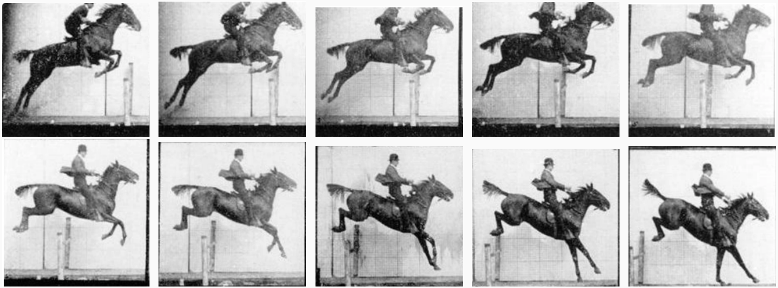

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Hobbs 1985: granularity relates to distinguishability

Hobbs 1985 on Granularity

Simplification

0	D_0	P_0	$R_0 \subseteq P_0$
1	$D_1 = D_0/\sim_{R_0}$	P_1	$R_1 \subseteq P_1$
2	$D_2 = D_1/\sim_{R_1}$	P_2	$R_2 \subseteq P_2$
...	...	...	...

$x \sim_R y$ iff $\forall P \in R[P(x) = P(y)]$

$[x]_\sim = \{y | x \sim y\}$

$D/\sim = \{[x]_\sim | x \in D\}$

Hobbs 1985 on Granularity

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Utrecht(5:20)
Maarsen(5:25)
Breukelen(5:30)
⋮
AmsterdamCS(6:00)
Start(5:20)
End(6:00)

Hobbs 1985 on Granularity

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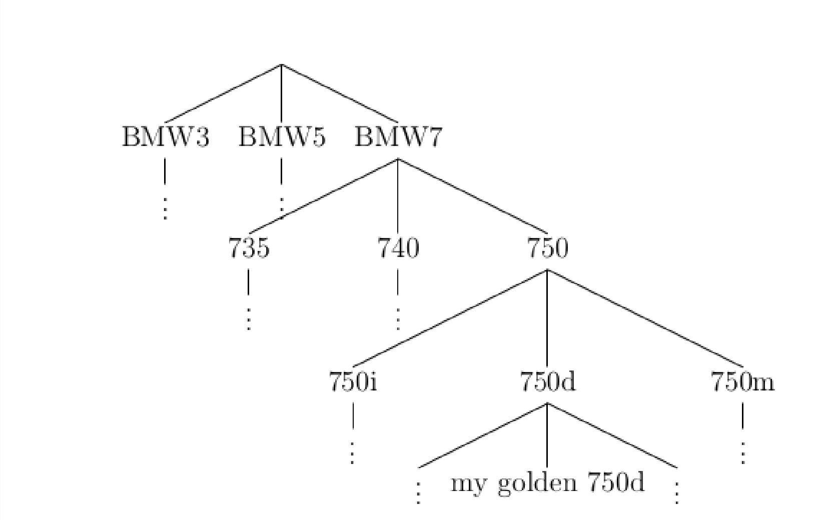
If $R = \{\text{Start, End}\}$, then 05:38$\sim_R$5:44.

$\mathrm{D}_1 = \{[05:20], (05:20, 06:00), [6:00]\}$

Granularity



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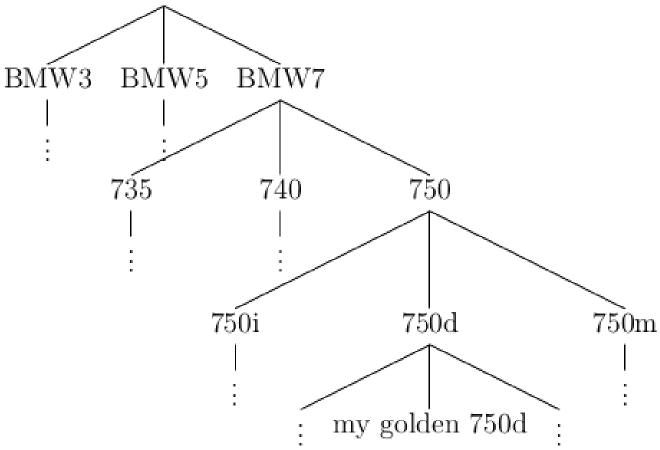
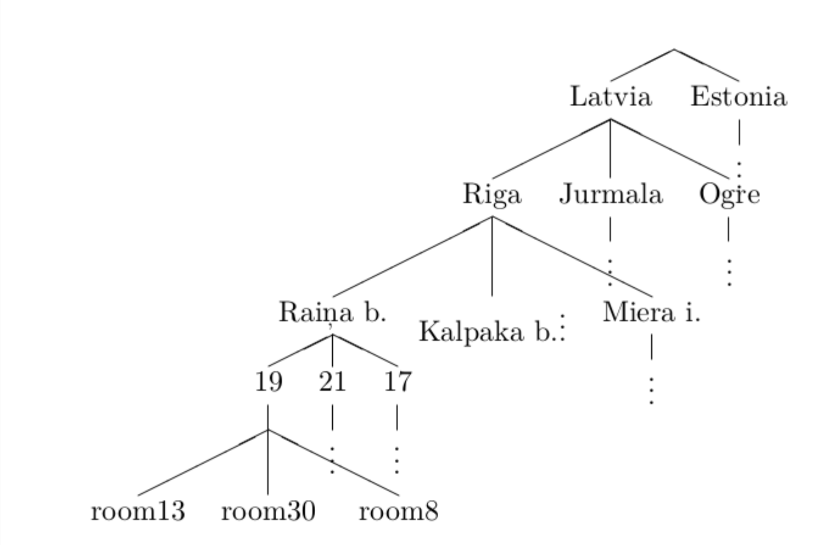


I like that car

Granularity



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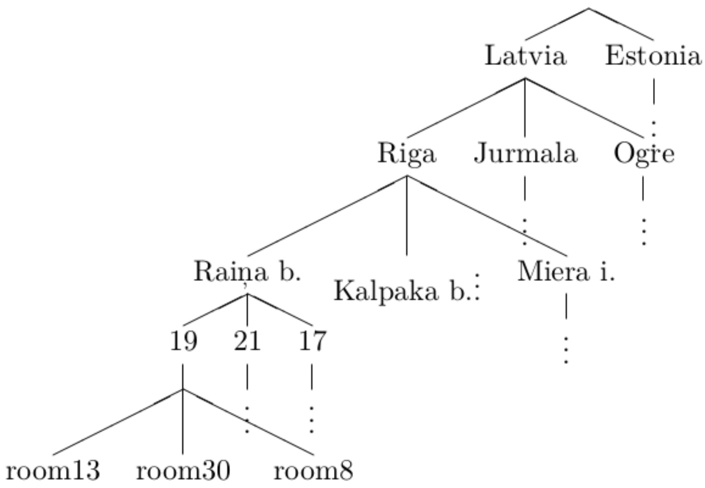
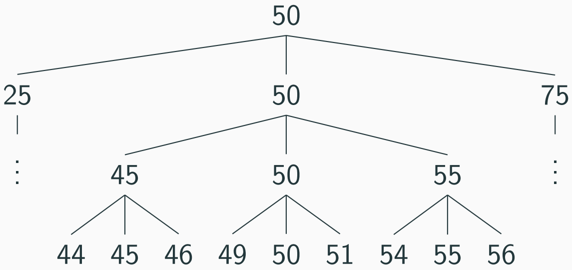


Juris was born **here**

Granularity in numbers



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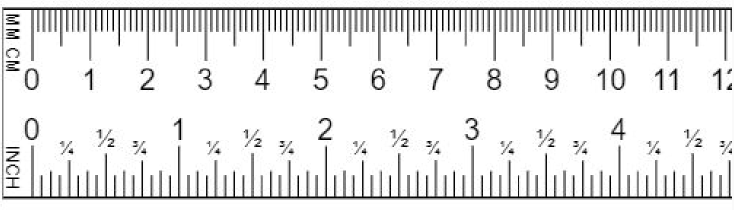

(Krifka 2007, Sauerland & Stateva 2010)

Formalizing granularity

A granularity function is a function $g_i : \mathbb{N} \to \mathbb{N}$ such that:

$$g_i(n) \;:=\; \text{the nearest multiple of } i \text{ to } n$$

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$g_5(46) = 45$ $\qquad$ $g_{10}(46) = 50$ $\qquad$ $g_{1000}(3960) = 4000$

Note that $g_1 \ = \ \mathrm{ID}$

Formalizing granularity

From now on, we assume granularity is a parameter for interpretation:

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Say $\#x = 46$ in w, then:

$[\![\#x = 50]\!]^{w,g_{10}} = 1$ $\qquad$ $[\![\#x = 50]\!]^{w,g_5} = 0$ $\qquad$ $[\![\#x = 50]\!]^{w,g_1} = 0$

Formalizing granularity: probabilistically

given a granularity level g, $P(w|u) = \frac{[\![u]\!]^{w,g}}{|\{\lambda w'.[\![u]\!]^{w',g}=1\}|}$

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Fifty students came to the party

	w_{47}	w_{48}	w_{49}	w_{50}	w_{51}	w_{52}	w_{53}
g_1	0	0	0	1	0	0	0
g_5	0	1	1	1	1	1	0
g_{10}	1	1	1	1	1	1	1

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g_1	0	0	0	1	0	0	0
g_5	0	.2	.2	.2	.2	.2	0
g_{10}	.11	.11	.11	.11	.11	.11	.11

Formalizing granularity: probabilistically

$$P(w, g_i | u) \propto \frac{[\![u]\!]^{w,g_i}}{|\{\lambda w'.[\![u]\!]^{w',g_i} = 1\}|} \times P(g_i)$$

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g_5	0	.1	.1	.1	.1	.1	0

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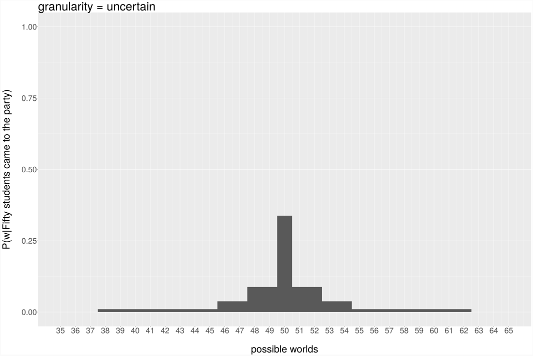

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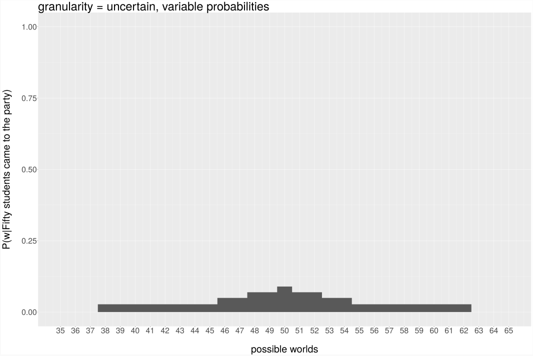

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Forty-nine students came to the party

	w_{47}	w_{48}	w_{49}	w_{50}	w_{51}	w_{52}	w_{53}
g_1	0	0	1	0	0	0	0
g_5	0	0	0	0	0	0	0

	w_{47}	w_{48}	w_{49}	w_{50}	w_{51}	w_{52}	w_{53}
g_1	0	0	1	0	0	0	0
g_5	0	0	0	0	0	0	0

A note on *exactly*

Exactly fifty students came to the party

- The presence of an upperbound
- Precision (fine granularity)
- Certainty?

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Context: the ESSLLI organization provides extra tickets to the party on Friday for participants who would like to take their children along. They only provide at most two extra tickets per participant. Will this be enough?

A note on *exactly*

Exactly fifty students came to the party

- The presence of an upperbound
- Precision (fine granularity)
- Certainty?

Context: the ESSLLI organization provides extra tickets to the party on Friday for participants who would like to take their children along. They only provide at most two extra tickets per participant. Will this be enough?

I have more than two children. (Nouwen 2010)
I have at least three children.

Vagueness regulation (Sauerland and Stateva 2010)

exactly fifty | approximately fifty | roughly fifty

Vagueness regulation (Sauerland and Stateva 2010)

exactly fifty approximately fifty roughly fifty

$$[\![\ldots \text{approximately } n \ldots]\!]^{g_i} = [\![\ldots n \ldots]\!]^{g_{i+\delta}}$$

Vagueness regulation (Sauerland and Stateva 2010)

exactly fifty   approximately fifty   roughly fifty

$[\![\ldots \text{approximately } n \ldots]\!]^{g_i} = [\![\ldots n \ldots]\!]^{g_{i+\delta}}$

$[\![\ldots \text{exactly } n \ldots]\!]^{g_i} = [\![\ldots n \ldots]\!]^{g_1}$

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Exactly / Approximately fifty students came to the party

disambiguation

Vagueness regulation (Sauerland and Stateva 2010)

exactly fifty   approximately fifty   roughly fifty

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$[\![\ldots \text{exactly } n \ldots]\!]^{g_i} = [\![\ldots n \ldots]\!]^{g_1}$

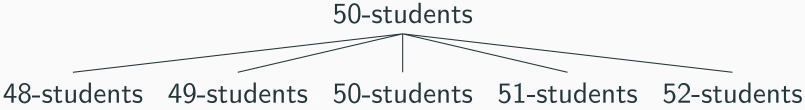
Exactly / Approximately fifty students came to the party

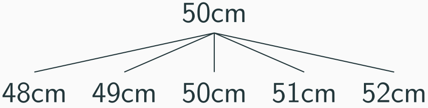
disambiguation

Exactly forty-nine students came to the party   prediction: vacuous

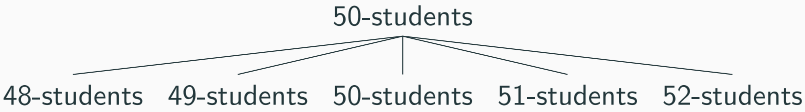
The rod was exactly forty-nine cm long   prediction: non-vacuous

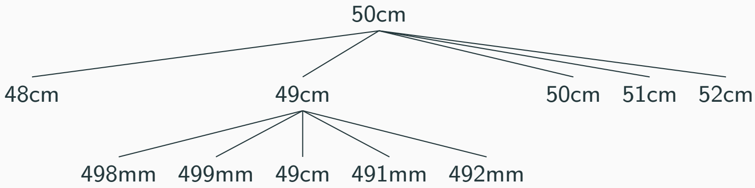
Discrete versus non-discrete domains





Discrete versus non-discrete domains





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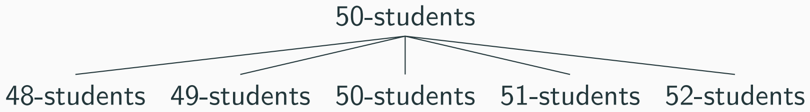
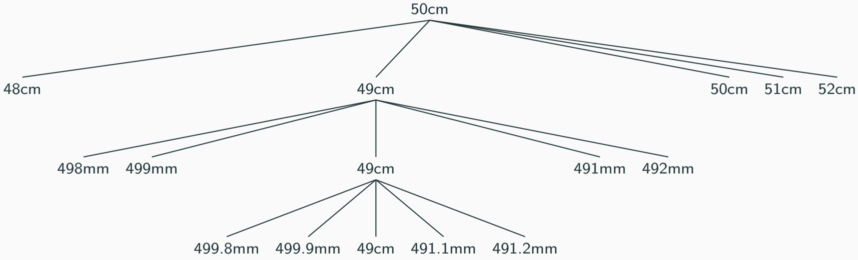
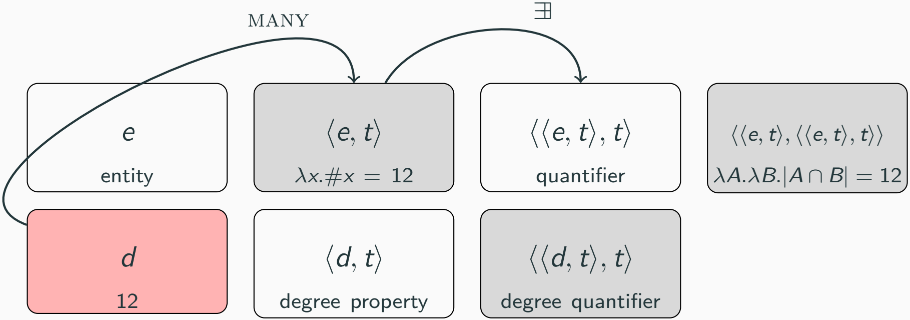




Filling in the details



Filling in the details

Fifty students came to the party

$$\exists x[\#x = 50 \wedge {}^{*}\text{student}(x) \wedge {}^{*}\text{came-to-the-party}(x)]$$
$$\wedge\ \neg\exists x[\#x > 50 \wedge {}^{*}\text{student}(x) \wedge {}^{*}\text{came-to-the-party}(x)]$$

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$$\wedge \neg\exists x[\#x > 50 \wedge {}^{*}\text{student}(x) \wedge {}^{*}\text{came-to-the-party}(x)]$$

In world w_{48} there are groups of students that came to the party of any cardinality up to 48.

Let x be the largest such cardinality, then $g_5(\#x) = 50$ and so, w.r.t. g_5 there is a group of relevant students of cardinality 50 in w_{48}.

Filling in the details

Fifty students came to the party

$$\exists x[\#x = 50 \wedge {}^*\text{student}(x) \wedge {}^*\text{came-to-the-party}(x)]$$
$$\wedge \neg\exists x[\#x > 50 \wedge {}^*\text{student}(x) \wedge {}^*\text{came-to-the-party}(x)]$$

In world w_{52} there are groups of students that came to the party of any cardinality up to 52.

Let x be the largest such cardinality, then in $g_5(\#x) = 50$ and so, w.r.t. g_5 there is a group of relevant students of cardinality 50 in w_{52} **while there is no group of relevant students of cardinality larger than 50**.

Filling in the details

Fifty students came to the party

$$\exists x[\#x = 50 \wedge^{*}\text{student}(x) \wedge^{*}\text{came-to-the-party}(x)]$$
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So, all is fine!

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$$max_d[\exists x[\# x = d \wedge^* \text{student}(x) \wedge^* \text{came-to-the-party}(x)]] \ = \ 50$$

Filling in the details

Fifty students came to the party

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In world w_{48} there are groups of students that came to the party of any cardinality up to 48.

In g_5 this means that we can find a group such that $\#x = 50$ (take the largest group), and so the interval in the scope of the maximality operator is [1,50].

Filling in the details

Fifty students came to the party

$max_d[\exists x[\#x = d \wedge^{*} \text{student}(x) \wedge^{*} \text{came-to-the-party}(x)]] \ = \ 50$

In world w_{52} there are groups of students that came to the party of any cardinality up to 48.

In g_5 this means that we can find a group such that $\#x = 50$ (take the largest group), but no larger group, and so the interval in the scope of the maximality operator is [1,50].

Filling in the details

Fifty students came to the party

$max_d[\exists x[\#x = d \wedge^* \text{student}(x) \wedge^* \text{came-to-the-party}(x)]] \; = \; 50$

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So all is fine!

A nice puzzle (Solt 2016)

(i) More than forty-nine students came to the party

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$[\![\ldots \#x > 49 \ldots]\!]^{w_{49}, g_5}$

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(i) More than forty-nine students came to the party

$$[\![\ldots \#x > 49 \ldots]\!]^{w_{49}, g_5} = 1$$

Beyond precision (Kao et al. 2014)

Hyperbole:

I have { ninety-eight / a hundred / five hundred million } unread emails in my inbox

$$P(w, g, \text{intention} | u)$$

Trade-off: obviously false utterances are not intended to provide objective information about the world

See you tomorrow!

`bit.ly/esslli-numsem`