Numeral Semantics | Tuesday

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`bit.ly/esslli-numsem`

Numeral Semantics

$$[\![\text{twelve}]\!]$$

Type landscape

e entity	$\langle e, t \rangle$ property	$\langle \langle e, t \rangle, t \rangle$ quantifier	$\langle \langle e, t \rangle, \langle \langle e, t \rangle, t \rangle \rangle$ quantifier
d degree	$\langle d, t \rangle$ degree property	$\langle \langle d, t \rangle, t \rangle$ degree quantifier	

The degree view

$[\![\text{twelve}]\!] = 12$

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$[\![\text{There are three times more A's than B's}]\!] = |A| = |B| \times 3$

The degree view: prenominal occurrences

(1) Twelve students came to the party.

We need to understand more about the meaning of nouns.

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$$[\![\text{student}]\!] \;=\; \lambda x.\mathit{student}(x)$$

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Semantic Plurality (e.g. Landman 1989, cf. Nouwen 2015)

$a \qquad b \qquad c \qquad d$

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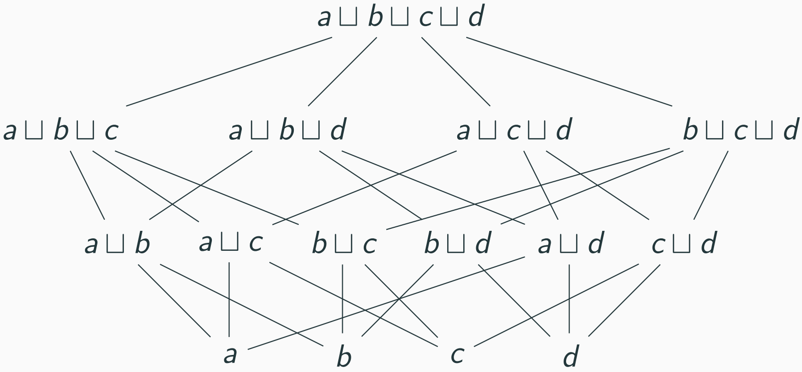

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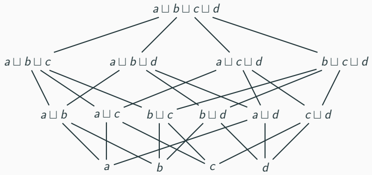

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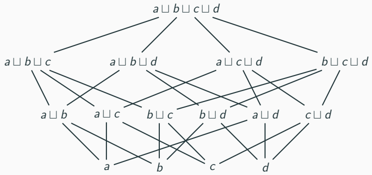

Let X be a set

$\sqcup X$:= the smallest entity x
such that each element of X is a part of x

$$^*X := \{\sqcup Y \mid Y \in \wp^+(X)\} = \{\sqcup Y \mid Y \subseteq X \;\&\; Y \neq \emptyset\}$$

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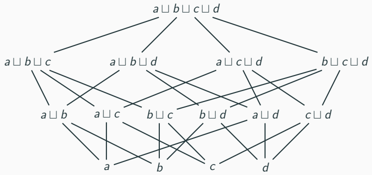

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$^*P(x)$ is true w.r.t. g and I iff $g(x) \in {}^*I(P)$

Number view

$$[\![\text{the}]\!] \;=\; \lambda P.\iota x.P(x) \;\wedge\; \neg\exists y[y \sqsupset x \;\wedge\; P(x)] \qquad \langle\langle e,t\rangle, e\rangle$$

$$[\![\text{students}]\!] = \lambda x.^{*}student(x) \qquad \langle e,t\rangle$$

$$[\![\text{the students}]\!] = \iota x.^{*}student(x) \wedge \neg\exists y[y \sqsupset x \wedge^{*} student(x)] \qquad e$$

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$$[\![\text{twelve}]\!] \;=\; 12 \qquad d$$

$$[\![\text{students}]\!] = \lambda x.{}^{*}student(x) \qquad \langle e,t\rangle$$

$$[\![\text{twelve students}]\!] = \qquad ?$$

Two routes

1. Keep type *d* denotation for numerals and provide a connection with nouns
2. Change the numeral denotation

Connecting numbers and nouns: Hackl 1999

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twelve students $\rightsquigarrow$ [[twelve MANY] students]

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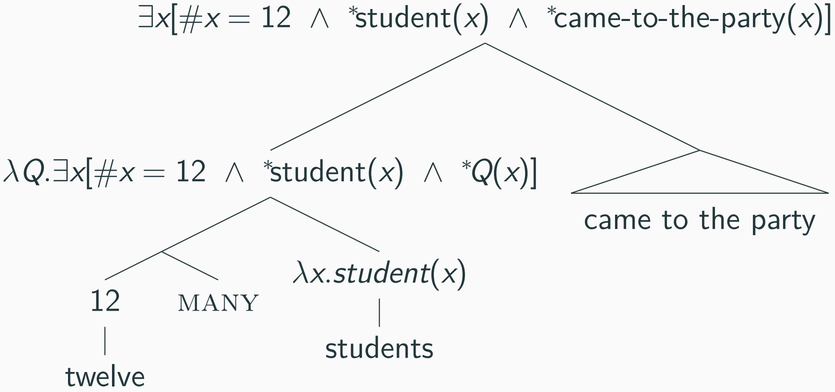
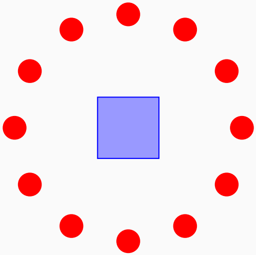
MANY

Connecting numbers and nouns



Connecting numbers and nouns: plurality at work

(6) In this picture, **twelve** dots surround the square.



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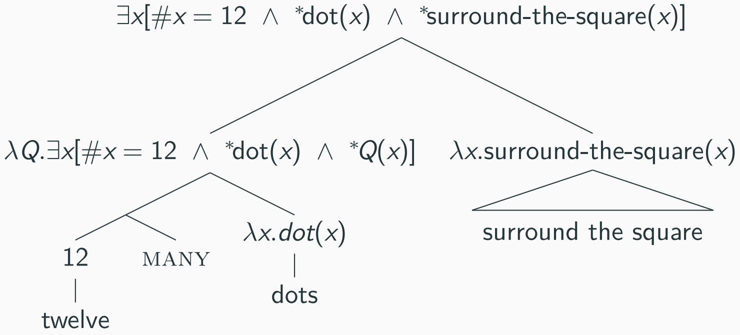

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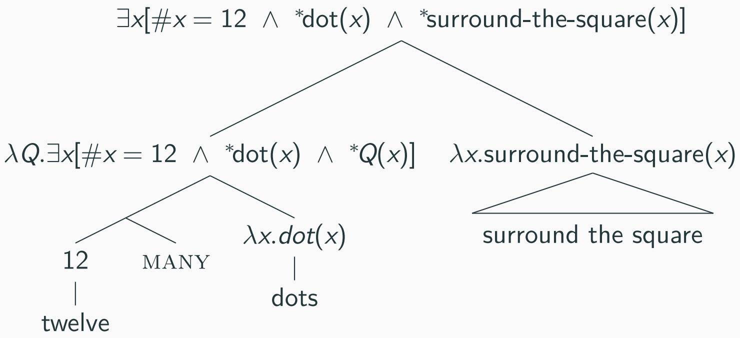

$\lambda x.\text{surround-the-square}(x) \rightsquigarrow \{c_1 \sqcup c_2 \sqcup \ldots \sqcup c_{12}\}$

$\lambda x.{}^{*}\text{surround-the-square}(x) \rightsquigarrow \{c_1 \sqcup c_2 \sqcup \ldots \sqcup c_{12}\}$

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(6) **Two** students carried the piano up the stairs.

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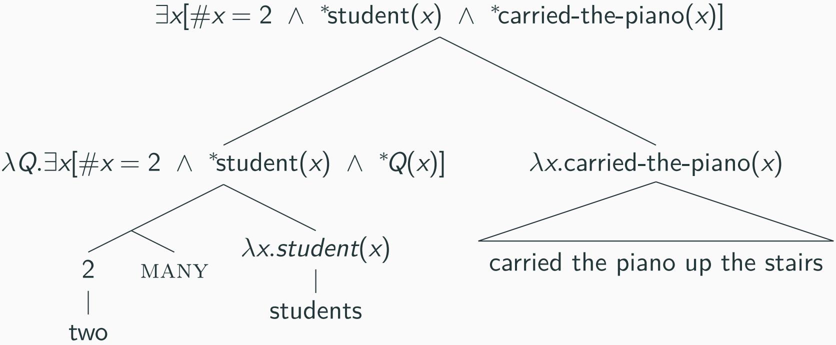

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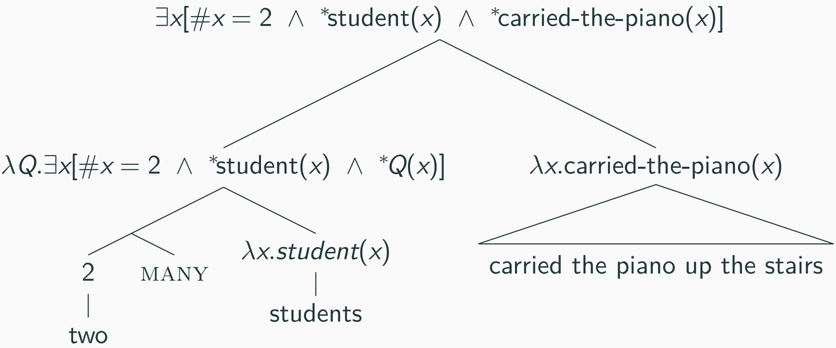

$\lambda x.\text{carry-the-piano}(x) \rightsquigarrow \{s_1, s_2\}$

$\lambda x.{}^{*}\text{carry-the-piano}(x) \rightsquigarrow \{s_1, s_2, s_1 \sqcup s_2\}$

Connecting numbers and nouns: plurality at work

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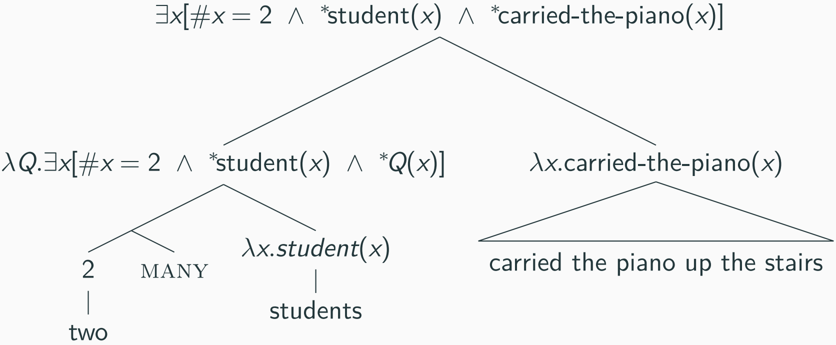

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Connecting numbers and nouns

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Connecting numbers and nouns

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(2) Every two houses come with one parking space.

(3) the twelve students

(4) Twelve people can fit in this lift.

Modificational MANY

$$[\![\text{MANY}]\!] = \lambda d.\lambda P.\lambda x.\#x = d \ \wedge \ {}^{*}P(x)$$

$$[\![\text{twelve MANY}]\!] = \lambda P.\lambda x.\#x = d \ \wedge \ {}^{*}P(x)$$

Buccola 2017; Blok, Bylinina & Nouwen 2019

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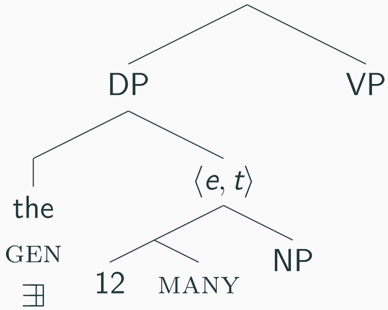

(5) The twelve students arrived. the

(6) Twelve people can fit in the lift. GEN

(7) Twelve students came to the party. ∃

Modificational MANY and existential closure

$$[\![\exists]\!] = \lambda A.\lambda B.\exists x[A(x) \wedge B(x)]$$

$$[\![\exists\ [\ [\text{twelve MANY}]\ \text{students}]]\!] = \lambda B.\exists x[\#x = 12 \ \wedge\ {}^{*}\text{student}(x) \ \wedge\ B(x)]$$

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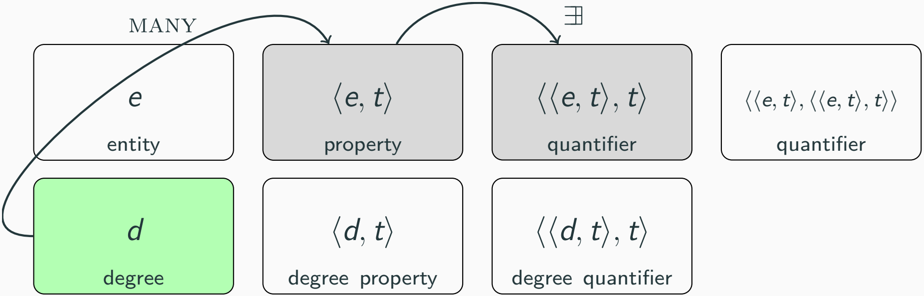

Aside: Connecting numbers and nouns

Krifka 1989:

$$[\![\text{students}]\!] = \lambda d.\lambda x.\#x = d \ \wedge \ {}^{*}student(x)$$

The number view $\sim$ the modifier view

MANY

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Modifier view: Reminder

$[\![\text{twelve}]\!] = \lambda x.\#x = 12$ (the set of groups of cardinality 12)

$[\![\text{American}]\!] = \lambda x.\mathit{american}(x)$ (the set of American entities)

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$\lambda x.\mathit{american}(x)$ $\lambda x.\mathit{student}(x)$

American student

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$$\lambda x.american(x) \wedge student(x)$$

$$\lambda x.american(x) \qquad \lambda x.student(x)$$

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twelve students

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twelve students

Modifier view across the board

$$[\![\text{twelve}_m]\!] \;=\; \lambda x.\#x = d$$

$$[\![\text{twelve}_d]\!] \;=\; 12$$

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$[\![\text{twelve}_m]\!] \; = \; \lambda x.\#x = d$

$[\![\text{twelve}_d]\!] \; = \; 12$

$[\![\textsc{many}]\!] \; = \; \lambda d.\lambda x.\#x = d$

Modifier view across the board

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$$[\![\textsc{many}]\!] \;=\; \lambda d.\lambda x.\#x = d$$

$$[\![\textsc{many}]\!]([\![\text{twelve}_d]\!]) = [\![\text{twelve}_m]\!]$$

Modifier view across the board

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$$[\![\text{twelve}_d]\!] \;=\; 12$$

$$[\![\textsc{many}]\!] \;=\; \lambda d.\lambda x.\#x = d$$

$$[\![\textsc{many}]\!]([\![\text{twelve}_d]\!]) = [\![\text{twelve}_m]\!]$$

$$[\![\textsc{card}]\!] = \lambda P.\iota d.\forall x[P(x) \rightarrow \#x = d]$$

Modifier view across the board

$$[\![\text{twelve}_m]\!] \; = \; \lambda x.\#x = d$$

$$[\![\text{twelve}_d]\!] \; = \; 12$$

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Summary so far

Two views:

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Crucial ingredients:

- Plurality
- External source of quantificational force:
  existential closure

Digression: Zero and plurality

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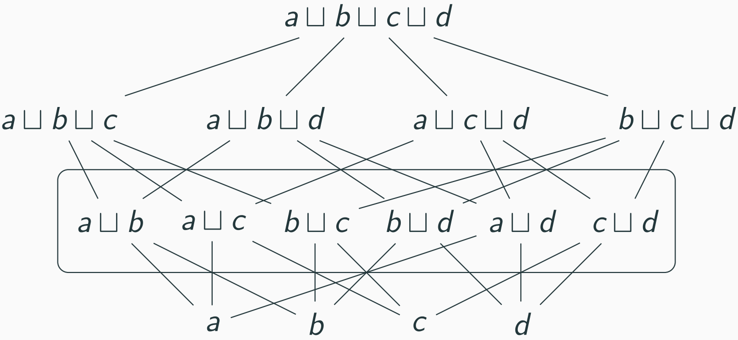
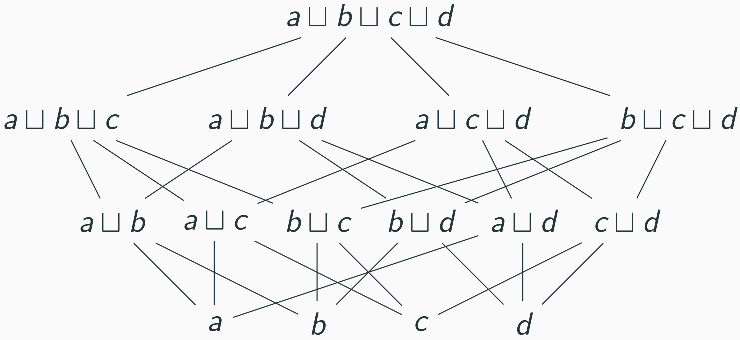


Digression: Zero and plurality

$$[\![\text{zero}_m]\!] = [\![\text{zero}_d \ \text{MANY}]\!] = \lambda x.\#x = 0$$



What does ‘zero cardinality’ mean?

Digression: Zero and plurality

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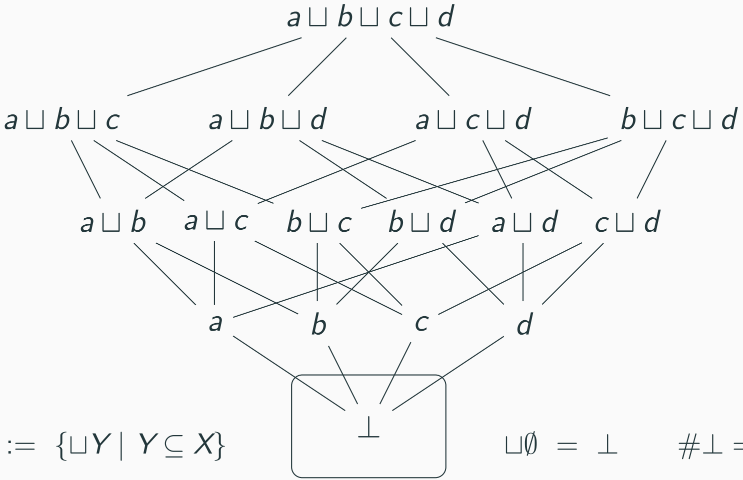

$^{*}X \ := \ \{\sqcup Y \mid Y \subseteq X\}$

$\sqcup\emptyset \ = \ \perp \qquad \#\perp = 0$

Digression: zero

- If we want to include zero in our semantics for numerals we need to include $\perp$ in our domain of plurals

- So let's assume the following:

  If P is true of any entity in X,
  then $^*P$ is true of any plurality formed by a subset of X

  This includes the plurality formed out of the empty set: $\perp$

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- This has drastic consequences...

Digression: zero

(i) There are women on this committee.

$\exists x[{}^{*}\text{woman}(x) \wedge {}^{*}\text{on-this-committee}(x)]$

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Let W be the set of women, then woman(x) is true if the value of x is in W

Now, $^{*}woman(x)$ is true if the value of x is a plurality that can be formed from the elements of some subset of W.

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Now, ${}^{*}woman(x)$ is true if the value of x is a plurality that can be formed from the elements of some subset of W.

But now (i) is predicted to be a tautology, just take $\perp$ for x.

Digression: zero

- The semantics of bare plurals would need to exclude $\perp$
- $\exists x[\#x > 0 \wedge {}^{*}\text{woman}(x) \wedge {}^{*}\text{on-this-committee}(x)]$

Digression: zero

- The semantics of bare plurals would need to exclude $\bot$
- $\exists x[\#x > 0 \wedge {}^*\text{woman}(x) \wedge {}^*\text{on-this-committee}(x)]$

- Further consequence: [the + plural noun] will in principle always refer

- Landman 2011 argues this is not a bad thing:

  Your honor, the persons who have come to me during 2004 with a winning lottery ticket have gotten a prize. Fortunately, I was on a polar expedition the whole year.

Back to where we were

Two views:

- **The degree view**: numerals as degrees
- **The modifier view**: numerals as properties
- Using a shift like MANY, these views collapse into one

Crucial ingredients:

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- External source of quantificational force: existential closure

Why have a degree slot?

- It's hard to argue for the *d*-slot for bare numerals
- Modified numerals provide some evidence

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- They can appear in a prenominal position

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more than twelve

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- They can appear in a prenominal position

- **Modifier view:** If (modified) numerals are type $\langle e, t\rangle$, they can be interpreted *in situ*
- **Degree view:** (Modified) numerals need to QR to resolve a type conflict (type d is expected)

Evidence for scope

(8) *Jan hoeft te scoren. NPI hoeven

Jan has to score

(9) Niemand hoeft te scoren.

nobody has to score

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(10) *Jan hoeft drie boeken te lezen.

Jan has three books to read

(11) Jan hoeft maximaal drie boeken te lezen.

Jan has maximally three books to read

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Explanation:

- maximaal drie is a quantifier over degrees
- Quantifiers take scope!

What about zero?

(10) *Jan hoeft drie boeken te lezen.
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What about zero?

(10) *Jan hoeft drie boeken te lezen.
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Preview: Quantificational analysis of numerals (Kennedy 2015)

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(10) *Jan hoeft drie boeken te lezen.

Jan has three books to read

(13) Jan hoeft nul boeken te lezen.

Jan has zero books to read

So apparently, bare numerals can take scope!

Preview: Quantificational analysis of numerals (Kennedy 2015)

e entity	$\langle e, t \rangle$ property	$\langle \langle e, t \rangle, t \rangle$ quantifier	$\langle \langle e, t \rangle, \langle \langle e, t \rangle, t \rangle \rangle$ quantifier
d 12	$\langle d, t \rangle$ degree property	$\langle \langle d, t \rangle, t \rangle$ degree quantifier	

See you tomorrow!

`bit.ly/esslli-numsem`