Numeral Semantics | Monday

Lisa Bylinina & Rick Nouwen

ESSLLI 2019

bit.ly/esslli-numsem

Numeral Semantics

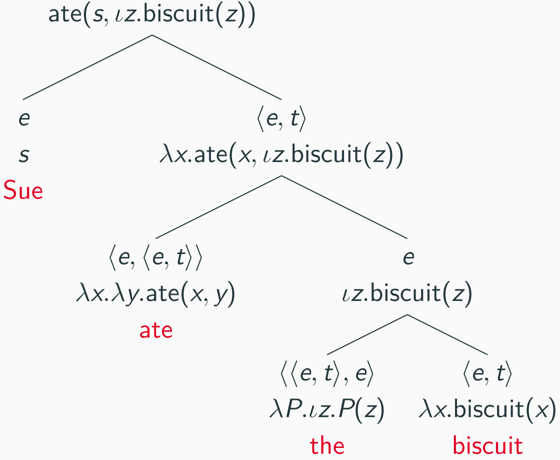
$$[\![\text{twelve}]\!]$$

Numeral Semantics

$[\![\text{twelve}]\!]$

$[\![\text{Twelve students came to the party}]\!]$

Required basic concept & Framework



- predicate logic
- typed lambda calculus

- Winter (LaLo-I)
- Glass (LaLo-I)
- Steinert-Threlkeld & Szymanik (LaCo-I)

Overview

Monday: Theories of numeral semantics

Tuesday: Continued

Wednesday: Continued

Thursday: (Im)precision

Friday: Beyond semantics

Numerals as determiners

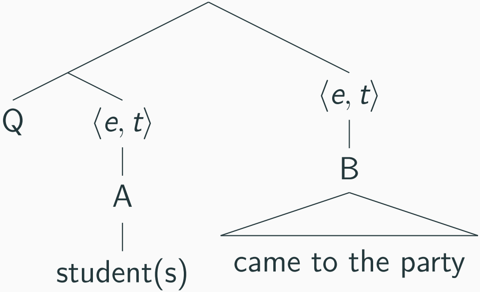
Twelve students came to the party.

Numerals as determiners

Twelve students came to the party.
Several students came to the party.
Some students came to the party.
Most students came to the party.
No students came to the party.

Generalized Quantifier Theory (Barwise & Cooper, 1980; Keenan & Stavi 1984)

Q student(s) came to the party

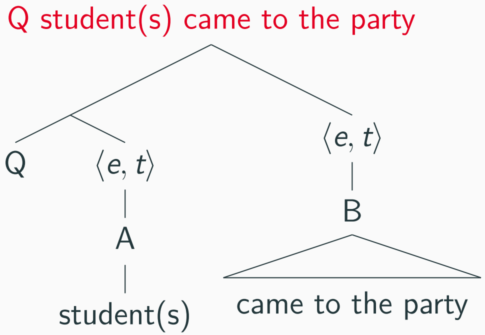


every(A)(B) iff $A \subseteq B$

no(A)(B) iff $A \cap B = \emptyset$

some(A)(B) iff $A \cap B \neq \emptyset$

Generalized Quantifier Theory (Barwise & Cooper, 1980; Keenan & Stavi 1984)



$$\begin{aligned} \text{every(A)(B)} &\quad \text{iff} \quad A \subseteq B \\ \text{no(A)(B)} &\quad \text{iff} \quad A \cap B = \emptyset \\ \text{some(A)(B)} &\quad \text{iff} \quad A \cap B \neq \emptyset \end{aligned}$$

Which of these are realised in NL?

What properties do all realised quantifiers share?

Constraints on quantifiers (van Benthem, 1984)

QUANT: $Q(A)(B) \Leftrightarrow Q(F[A])(F[B])$ for any bijection F

Constraints on quantifiers (van Benthem, 1984)

QUANT: $Q(A)(B) \Leftrightarrow Q(F[A])(F[B])$ for any bijection F

Upshot: only cardinalities ever matter

$$\begin{aligned} \text{every}(A)(B) &\quad \text{iff} \quad |A \cap B| = |A| \\ \text{no}(A)(B) &\quad \text{iff} \quad |A \cap B| = 0 \\ \text{some}(A)(B) &\quad \text{iff} \quad |A \cap B| \neq 0 \end{aligned}$$

Constraints on quantifiers (van Benthem, 1984)

QUANT: $Q(A)(B) \Leftrightarrow Q(F[A])(F[B])$ for any bijection F

Upshot: only cardinalities ever matter

every(A)(B) iff $|A \cap B| = |A|$
no(A)(B) iff $|A \cap B| = 0$
some(A)(B) iff $|A \cap B| \neq 0$

twelve(A)(B) iff $|A \cap B| = 12$

Numerals are not quantifiers

Numerals are not quantifiers

(1) **Twelve** students came to the party.

(2) **Twelve** people can fit in the lift.

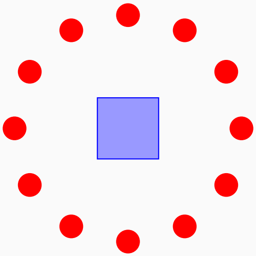
Numerals are not quantifiers

(3) **Twelve** of my students built a boat together.

(4) ***Every** student of mine built a boat together.

(5) ***Most** of my students built a boat together.

Numerals are not quantifiers



(6) In this picture, **twelve** dots surround the square.

(7) ??In this picture, **every** dot surrounds the square.

Numerals are not quantifiers

(8) This house has **twelve** windows.

(9) *This house has **every / most** window(s).

Numerals are not quantifiers

(10) The meeting lasted **twelve** hours.

(11) *The meeting lasted **most / every** hour(s).

Numerals are not quantifiers

(12) The girls in this class are **12** of our most promising students.

Numerals are not quantifiers

(12) The girls in this class are **12** of our most promising students.

(13) *The girls in this class are **every one** of our most promising students.

Numerals are not quantifiers

(14) Every **two** houses come with one parking space.

Numerals are not quantifiers

(15) Rod A is **three** times longer than rod B.

(16) **Two** is a Fibonacci number.

Three strands of thought

1. the number view

$[\![\text{twelve}]\!] \ = \ 12$

2. the modifier view

$[\![\text{twelve}]\!] \ = \ \lambda x.\#x = 12$

3. the quantifier view (revisited)

$[\![\text{twelve}]\!] \ = \ \text{the set of intervals that end in 12}$

The number view

$[\![\text{twelve}]\!] = 12$ whatever that means

The number view

$[\![\text{twelve}]\!] \; = \; 12$ whatever that means

(16) **Two** is a Fibonacci number.

$[\![\text{is a Fibonacci number}]\!] = \{0, 1, 2, 3, 5, 8, 13, 21, 34, \ldots\}$

The number view

$[\![\text{twelve}]\!] = 12$ whatever that means

(15') Rod A is longer than rod B.

means: the length of rod A $>$ the length of rod B

(15) Rod A is **three** times longer than rod B.

means: the length of rod A $= 3 \times$ the length of rod B

What could a number be?

Basic semantic ontology:

- Entities; type e
- Truth-values; type t

What could a number be?

Basic semantic ontology:

- Entities; type e
- Truth-values; type t

We are going to add **degrees** to this picture: type d

- Degrees are like entities, but ordered

$$\text{John} < \text{Mary} \qquad 2 < 3$$

- Numbers are a special case of degrees

What could a degree be?

(17) John is taller than Mary.

What could a degree be?

(17) John is taller than Mary.

John's height $>$ Mary's height

What could a degree be?

(17) John is taller than Mary.
John's height $>$ Mary's height

(18) John is 2cm taller than Mary.

What could a degree be?

(17) John is taller than Mary.
John's height $>$ Mary's height

(18) John is 2cm taller than Mary.
John's height $=$ Mary's height $+$ 2cm

What could a degree be?

(17) John is taller than Mary.
John's height $>$ Mary's height

(18) John is 2cm taller than Mary.
John's height $=$ Mary's height $+$ 2cm

(19) John is this tall.

What could a degree be?

(17) John is taller than Mary.

John's height $>$ Mary's height

(18) John is 2cm taller than Mary.

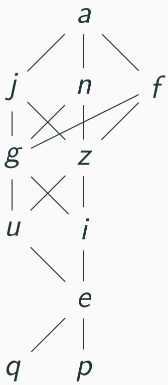
John's height $=$ Mary's height $+$ 2cm

(19) John is this tall.

this demonstrates a degree

What could a degree be?

- We seem to be committed to a domain of abstract entities
- We don't have to be:
  - Cresswell 1977: tall involves an ability to decide that $j < m$
  - We can see degrees as equivalence classes of individuals



Would could a degree be?

- This is an ordinal scale
- No distances, no zero, no multiplication
- Height etc. would need to be added to the equivalence classes (Bale 2011)
- Back to square one

The number view: challenges

How does this meaning connect with nouns, to give us sets with particular cardinalities?

(1) Twelve students came to the party.

(2) Twelve people can fit in the lift.

(20) Every two houses come with one parking space.

The number view: challenges

How does this meaning connect with nouns, to give us sets with particular cardinalities?

(1) Twelve students came to the party.

(2) Twelve people can fit in the lift.

(20) Every two houses come with one parking space.

Preview of options:

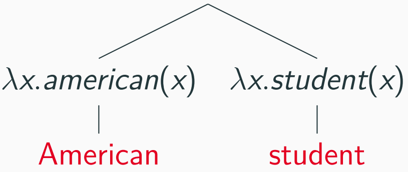
1. Keep this meaning of numerals, add something that connects it with nouns (lecture 2)
2. Assume a different meaning of numerals (in this position)

The modifier view

The modifier view

$[\![\text{twelve}]\!] = \lambda x.\#x = 12$ (the set of groups of cardinality 12)

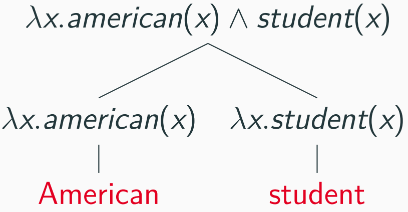
$[\![\text{American}]\!] = \lambda x.\mathit{american}(x)$ (the set of American entities)



The modifier view

$[\![\text{twelve}]\!] = \lambda x.\#x = 12$ (the set of groups of cardinality 12)

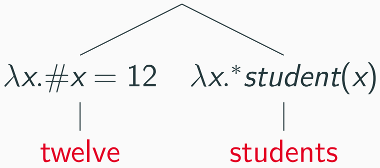
$[\![\text{American}]\!] = \lambda x.american(x)$ (the set of American entities)



The modifier view

$[\![\text{twelve}]\!] = \lambda x.\#x = 12$ (the set of groups of cardinality 12)

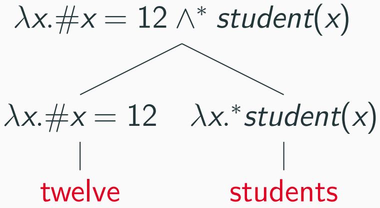
$[\![\text{American}]\!] = \lambda x.american(x)$ (the set of American entities)



The modifier view

$[\![\text{twelve}]\!] = \lambda x.\#x = 12$ (the set of groups of cardinality 12)

$[\![\text{American}]\!] = \lambda x.\mathit{american}(x)$ (the set of American entities)



The modifier view

$[\![\text{twelve}]\!] = \lambda x.\#x = 12$ (the set of groups of cardinality 12)

$[\![\text{American}]\!] = \lambda x.american(x)$ (the set of American entities)

every twelve students now parallels every American student

The modifier view

$[\![\text{twelve}]\!] = \lambda x.\#x = 12$ (the set of groups of cardinality 12)

$[\![\text{American}]\!] = \lambda x.\mathit{american}(x)$ (the set of American entities)

every twelve students now parallels every American student

The numeral expresses cardinality, not quantificational force

The modifier view

In the absence of a determiner, it's parallel to a bare plural:

(1) Twelve students came to the party.

(21) American students came to the party.

The modifier view

In the absence of a determiner, it's parallel to a bare plural:

(1) Twelve students came to the party.

(21) American students came to the party.

We will return to this.

The modifier view: challenges

The modifier view: challenges

(16) **Two** is a Fibonacci number.

The modifier view: challenges

(16) **Two** is a Fibonacci number.

This is not NP ellipsis:

(16') ***Two** are a Fibonacci number.

(16'') **Two** boxes are/*is open.

The modifier view: challenges

(16) **Two** is a Fibonacci number.

This is not NP ellipsis:

(16') ***Two** are a Fibonacci number.

(16'') **Two** boxes are/*is open.

Maybe it's not a problem:

$[\![\text{is a Fibonacci number}]\!] =$
$\{\lambda x.\#x = 1,\ \lambda x.\#x = 2,\ \lambda x.\#x = 3, \lambda x.\#x = 5, \ldots\}$

Summary: Type landscape

e entity	$\langle e, t \rangle$ property	$\langle \langle e, t \rangle, t \rangle$ quantifier	$\langle \langle e, t \rangle, \langle \langle e, t \rangle, t \rangle \rangle$ quantifier
d degree	$\langle d, t \rangle$ degree property	$\langle \langle d, t \rangle, t \rangle$ degree quantifier	

Summary: Type landscape

e entity	$\langle e, t\rangle$ property	$\langle\langle e, t\rangle, t\rangle$ quantifier	$\langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle\rangle$ quantifier
d degree	$\langle d, t\rangle$ degree property	$\langle\langle d, t\rangle, t\rangle$ degree quantifier	

Summary: Type landscape

e entity	$\langle e, t \rangle$ property	$\langle\langle e, t \rangle, t \rangle$ quantifier	$\langle\langle e, t \rangle, \langle\langle e, t \rangle, t \rangle\rangle$ quantifier
d degree	$\langle d, t \rangle$ degree property	$\langle\langle d, t \rangle, t \rangle$ degree quantifier	

Summary: Type landscape

e entity	$\langle e, t\rangle$ property	$\langle\langle e, t\rangle, t\rangle$ quantifier	$\langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle\rangle$ quantifier
d degree	$\langle d, t\rangle$ degree property	$\langle\langle d, t\rangle, t\rangle$ degree quantifier	

Summary: Type landscape

e entity	$\langle e, t\rangle$ property	$\langle\langle e, t\rangle, t\rangle$ quantifier	$\langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle\rangle$ quantifier
d degree	$\langle d, t\rangle$ degree property	$\langle\langle d, t\rangle, t\rangle$ degree quantifier	

See you tomorrow!